RESPONSE-BASED STATISTICS
Part 2

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ENVIRONMENTAL CONTOURS
How to estimate N-Year Wind/Wave Design Loads?
Deep Water LRFD Design

Load = L (Hs, Tp )

Seek 100-yr contour \([Hs, Tp]_{100}\) so that

\[ L_{100} = \text{worst load along } [Hs, Tp]_{100} \]

How to construct \([Hs, Tp]_{100}\) ?

Wind: Interest in \(L_{50}\)… Similar contours of \(V=\text{mean wind speed, } I=\text{turbulence intensity}\)
The Problem and FORM solution

- Load = \( Y(H_s, T_p) \rightarrow Y(U_1, U_2) \) where \( U_1, U_2 \) are standard normal
The Problem and FORM solution

- Load = $Y(Hs, Tp)$ $\rightarrow$ $Y(U_1, U_2)$ where $U_1$, $U_2$ are standard normal

- Failure if $Y > y_{\text{CAP}}$ $\rightarrow$ $M = y_{\text{CAP}} - Y(U_1, U_2) < 0$
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- Failure if \( Y > y_{\text{CAP}} \) \( \rightarrow M = y_{\text{CAP}} - Y(U_1, U_2) < 0 \)

- Given \( y_{\text{CAP}} \) \( \rightarrow p_F = P[y_{\text{CAP}} - Y(U_1, U_2) < 0] \)
  estimated with FORM
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• Given \( y_{\text{CAP}} \rightarrow p_F = P[y_{\text{CAP}} - Y(U_1, U_2) < 0 ] \) estimated with FORM

• Given \( p_F = p_{F,\text{TARGET}} \) (e.g., \( 10^{-2} / \text{year} \)) \rightarrow \) want consistent \( y_{\text{CAP}} \) (e.g., \( y_{100} \))
Brute Force
FORM
solution
Brute Force FORM solution

- Assume $y_{\text{CAP}} = y_1$
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- FORM: find $\beta_1$, $p_{F1} = \Phi(-\beta_1)$
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- Suppose $p_{F1} < p_{F,\text{TARGET}}$
- Reduce $y_{\text{CAP}}$ from $y_1$ to $y_2$
**Brute Force FORM solution**

- Assume $y_{\text{CAP}} = y_1$
- FORM: find $\beta_1$, $p_{F1} = \Phi(-\beta_1)$
- Suppose $p_{F1} < p_{F,\text{TARGET}}$
- **Reduce** $y_{\text{CAP}}$ from $y_1$ to $y_2$
- FORM: find $\beta_2$, $p_{F2} = \Phi(-\beta_2)$
Brute Force FORM solution

- Assume $y_{\text{CAP}} = y_1$
- FORM: find $\beta_1$, $p_{F1} = \Phi(-\beta_1)$
- Suppose $p_{F1} < p_{F,\text{TARGET}}$
- Reduce $y_{\text{CAP}}$ from $y_1$ to $y_2$
- FORM: find $\beta_2$, $p_{F2} = \Phi(-\beta_2)$
- Suppose $p_{F2} > p_{F,\text{TARGET}}$
- Increase $y_{\text{CAP}}$ from $y_2$ to $y_3$
Brute Force FORM solution

• Iterate until we find $y_{\text{CAP}}$ for which $\beta = \text{target value}$ (here, $\beta = 3.7$)
Brute Force FORM solution

- **Iterate until we find** $y_{\text{CAP}}$ **for which** $\beta = \text{target value}$ (here, $\beta = 3.7$)

- **Observation:** This is silly
Brute Force FORM solution

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- Observation: This is silly

- Better: just search circular contour, with $|u| = \beta$, to find maximum value of load/response $Y(U_1, U_2)$
Brute Force FORM solution

- Iterate until we find $y_{\text{CAP}}$ for which $\beta =$ target value (here, $\beta = 3.7$)
- Observation: This is silly
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- Result: Environmental Contours
Contours 1: What are They?

Four sites considered (Winterstein et al 1999):

- **GOM1**: Generic GOM site (API TLP code calibration JIP)
- **GOM2**: Sudden GOM storms (smaller h100, can’t evacuate)
- **Southern NS**: Ekofisk
- **Northern NS**: Statfjord

Our base case shown here: GOM1 \(\Rightarrow H_{100}=11.7\) m
Contours 1: What are They?

Gulf of Mexico site:
\( H_{100} = 11.7 \text{m} \)

100-yr PDF contour: \( P[ \text{fall outside } ] = 0.01 / \text{yr} \)
Contours 1: What are They?

Gulf of Mexico site: \( H_{100} = 11.7 \text{m} \)

100-yr PDF contour: \( P[ \text{fall outside} ] = 0.01 / \text{yr} \)

100-yr FORM contour: \( P[\text{fall outside any tangent line}] = 0.01 / \text{yr} \)

So: 100-yr FORM contour includes \( H_{100} = 11.7 \text{m} \)
Contours 2: How to Draw Them?

• Start with Standard Normal Variables \((U_1, U_2)\)

• From target \(p_f\), get corresponding normal fractile \(\beta = \Phi^{-1}(1-p_F)\)

• Contour = circle with radius \(\beta\) (sphere if \(n=3\) vars)
Contours 2: How to Draw Them?

<table>
<thead>
<tr>
<th>Return Period $T$ [yrs]</th>
<th>Failure Probability $p_f$ [per 3-hr seastate]</th>
<th>Limit State Distance $\beta_T = \Phi^{-1}(1 - p_f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>$1.14 \times 10^{-5}$</td>
<td>4.24</td>
</tr>
<tr>
<td>100</td>
<td>$3.43 \times 10^{-6}$</td>
<td>4.50</td>
</tr>
<tr>
<td>300</td>
<td>$1.14 \times 10^{-6}$</td>
<td>4.73</td>
</tr>
<tr>
<td>1000</td>
<td>$3.43 \times 10^{-7}$</td>
<td>4.97</td>
</tr>
<tr>
<td>3000</td>
<td>$1.14 \times 10^{-7}$</td>
<td>5.17</td>
</tr>
<tr>
<td>10000</td>
<td>$3.43 \times 10^{-8}$</td>
<td>5.40</td>
</tr>
</tbody>
</table>
Contours 3: From U to X space

Plus:
Rosenblatt Transformation

\[ F_{H_s}(h) = \Phi(u_1) \]
\[ F_{T_p|H_s}(t|h) = \Phi(u_2) \]

Yields:
Contours 4: Including Response Variability

**PROBLEM**

\[ X_{100} = 100\text{-Year Load/Response is NOT SAME as} \]

Expected Load in Worst 100-Year Environment
(e.g., worst median \( X_{50} \) along \([H_s, T_p]_{100}\))

**SOLUTIONS**

1. **Exact**: Construct, search 3-D contour \([H_s, T_p, X]_{100}\)

2. **Approximate**:
   - Search \([H_s, T_p]_{100}\) for worst higher fractile load \( X_p \)
   - Calibration to exact results suggests \( p = 0.85-0.90 \)
Predicting 100-Year Wave Crests: the 85% solution works...

Fig. 7 Crest height extremes along the $10^{-2}$ – probability contour line.
EXAMPLE: SURGE MOTION OF A SPAR

- **SPAR GEOMETRY**: “Consensus Spar” established by OTRC

- **FORCES**: Linear and quadratic transfer functions (LTFs and QTFs) found for 6DOF from diffraction analysis (OTRC)

- **MOTIONS**: LTFs and QTFs found for surge motion

- **STATISTICS**: First 4 moments of surge found → Hermite model for extremes
EXAMPLE: SURGE MOTION OF A SPAR

• RESULT: Isolines of constant surge [m] as a function of $H_s$ and $T_p$
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- RESULT: Isolines of constant surge [m] as a function of $H_s$ and $T_p$
- NEED: worst value along 100-yr contour $\rightarrow y_{100}$ about 16m
- NOTE: don’t need isolines; just search contour (any response)
EXAMPLE: SURGE MOTION OF A SPAR

- **RESULT**: Isolines of constant surge [m] as a function of $H_S$ and $T_P$
- **NEED**: worst value along 100-yr contour $\rightarrow$ $y_{100}$ about 16m
- **IF**: We applied only $H_{100} = 11.7$ m, we’d get $y$ of about 14m
SURGE MOTION: SPAR VS TLP

- **ENVIRONMENT**: North Sea site; storms above $H_s=8\text{m}$
SURGE MOTION: SPAR VS TLP

- **ENVIRONMENT**: North Sea site; storms above $H_s=8m$
- **ISOLINES**: TLP especially period-sensitive ($T_p$ around 10s)

![Graph showing surge motion comparison between SPAR and TLP](image)
SURGE MOTION: SPAR VS TLP

- **ENVIRONMENT**: North Sea site; storms above $H_s = 8$ m
- **ISOLINES**: TLP especially period-sensitive ($T_p$ around 10s)
- **CONTOURS**: Depend only on environment (same in both cases)
HS-TP CONTOURS AT OTHER SITES

Contours generated by HTCNTR, 100 year return period

- Northern North Seas (1)
- Sleipner (9)
- Tromsoeflaket (10)
- Statfjord/Brent (11)
Contour generated by HTCNTR, 100 year return period (Event models)

- Statfjord event (3)
- Gulf of Mexico (4)
- Ekofisk event (5)
HS-TP CONTOURS AT OTHER SITES

Contours generated by HTCNTR, 100 year return period

- Ekofisk, new (12)
- Ekofisk, 1993 (2)

Spectral Peak period (T_p) in s

Significant Wave height (H_s) in m
HS-TP CONTOURS AT OTHER SITES

Contours generated by HTCNTR, 100 year return period

- Aasgard, all seasons (6)
- Aasgard, summer (4 months) (7)
- Aasgard, spring (2 months) (8)

Spectral Peak period (Tp) in s

Significant Wave height (Hs) in m

0  2  4  6  8  10  12  14  16
0  5  10  15  20  25  30
LRFD Study: Gulf of Mexico vs North Sea

- GOM1: Generic GOM site (API TLP code calibration JIP)
- GOM2: Sudden GOM storms (smaller h100, can’t evacuate)
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LRFD Study: Gulf of Mexico vs North Sea

• GOM environment relatively harsher above design (100-yr) conditions
• Load factor $1.3 \times L_{100}$ gives $pf$ about $10^{-3}$ (GOM)
  
  \[
  \ldots \ldots \ldots \ldots 10^{-4} \quad \text{(NS)}
  \]
Higher-Dimension Contours

• Include (scalar) current or response variability $\rightarrow n=3$

• Include directional effects ($n = 3$ or more)

• No longer easy to visualize as in 2D

• Still easy to construct/search for worst (N-year) load/response

• Optimization problem easier here (inverse FORM) than in standard FORM: need only to find optimum given “box-like” constraints in $n-1$ direction cosines ($n-1$ angles between 0 and $2\pi$). Both optimizations (FORM and inverse FORM) available in MATLAB.
Environmental Contours: Summary

• Easy to construct

• Not impossible to understand (FORM knowledge helps)

• Useful for structures sensitive to multiple environmental parameters (floating structures/ships, wind turbines)

• Convenient basis for simulation or experimental studies

• **Wave Examples:** Hs-Tp, Hs-Current, Hs-Wind Speed

• **Wind Examples:** Mean Wind Speed, Turbulence Intensity

• **Response Variability:** Choose higher fractile $X_p$ ($p=.85-.90$)
Contour References


