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Environmental Parameters for Extreme Response: Inverse FORM With Omission Factors

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Abstract

In structural reliability problems there is generally uncertainty both in the gross load environment, and in the extreme response given the loading. We show here how these uncertainties can be approximately decoupled. We find contours of environmental parameters along which specified extreme fractiles lie (e.g., 100-year values of any structural response quantity). These contours are independent of the structure, making them a practical way to display a two (or higher) dimensional environmental hazard at a site.

Based on the first-order reliability method (FORM), the inverse-FORM method is introduced. This searches a hypersphere of constant radius β to find the maximum response. FORM omission factors are used to permit correct results based on only the *median* response, which may be estimated either analytically or by simulation.

Applications to various offshore structure problems are shown, including prediction of extreme wave crests and the base shear of a shallow-water jacket structure. Results are found to compare well with full FORM analysis.

Introduction

In practical structural reliability problems there is generally uncertainty both in the gross load environment, and in the extreme dynamic response given the loading. Denoting the environmental variables by $\boldsymbol{X}=[X_1 \dots\ X_n]$ and the response by Y, the failure probability p_F can be written formally as

$$p_F = \int_{all\,\boldsymbol{x}} P[Y > y_{cap}|\boldsymbol{X} = \boldsymbol{x}] f(\boldsymbol{x}) d\boldsymbol{x} \tag{1}$$

In principle Eq. 1 can be estimated with FORM/SORM or simulation. Several practical difficulties may arise, however, which motivate this paper. First, Eq. 1 requires a full, coupled environment-response model; i.e., the joint description $f(\boldsymbol{x})$ of all environmental variables and the conditional failure probability, $P[Y > y_{cap}|\boldsymbol{X} = \boldsymbol{x}]$, for all $\boldsymbol{x}$.

For the structural analyst, however, it is simpler to require that only a limited set of environmental conditions (values of $\boldsymbol{x}$) be checked to ensure adequate capacity y_{cap}. For example, many shallow-water ocean structures are most sensitive to the significant wave height, H_S=$4\sigma_\eta$, in which σ_η is the wave elevation rms over a stationary "seastate." To estimate 100-year responses, one may then simply apply a seastate with 100-year H_S level, and representative values of other variables such as wave period, current, etc.:

$$X_1 = H_S = H_{100}\,; \quad X_i = \text{Median}[X_i|X_1 = H_{100}]\,,\ i = 2, 3, ... \tag{2}$$

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This also simplifies the task of the environmental analyst, who need only report parameters of this H_S-driven 100-year seastate.

More generally, however, the critical environment will be structure-dependent. For example, deeper-water structures may be affected by seastates with smaller H_S but larger current U, or with resonant values of peak spectral period T_P. This leads to interest in defining joint contours (e.g., H_S–T_P or H_S-U contours) along which 100-year levels of any deterministic response quantity will lie (e.g., Haver, 1987). This again decouples the environmental description from the specific structural design concept. By introducing the "inverse FORM" method, we show here how such contours can be directly generated. These contours, unlike some others proposed, contain Eq. 2 as a special case.

A further complication is that the foregoing approach—Eq. 2 and its generalizations—ignore uncertainty in the response Y given the environment $\boldsymbol{X}$. By using a representative Y, such as its median $y(\boldsymbol{X})$ given $\boldsymbol{X}$, extreme response fractiles may be underestimated. Again we may resort to Eq. 1, but we lose the benefit of the decoupled, environmental contour. Also, for nonlinear stochastic response, it may be expensive to accurately estimate the full distribution of Y given $\boldsymbol{X}$.

To overcome these obstacles, we introduce "inflated" environmental contours. These are based on FORM omission factors (Madsen, 1988). These determine how much the environmental contour return period should be inflated (e.g., from 100 to 200–300 years or more) to compensate for omitting uncertainty in the 100-year response given $\boldsymbol{X}$.

Environmental Contours for Deterministic Response

We first consider the response Y as a deterministic function of the seastate variables $\boldsymbol{X}$. For FORM purposes we transform $\boldsymbol{X}$ to a set of standard normal variables $\boldsymbol{U}$ (e.g., Madsen et al, 1986), so that

$$Y = y(\boldsymbol{U}) \tag{3}$$

In this case uncertainty in the environment (randomness in $\boldsymbol{X}$) is assumed to dominate, so that conditional uncertainty in Y given $\boldsymbol{X}$ (or given $\boldsymbol{U}$) is negligible. We generalize these results in the next section to include conditional uncertainty in Y as well.

The "Forward" FORM problem typically seeks the failure probability, p_F, associated with exceeding a known response capacity, y_{cap}. The FORM estimate of p_F, and associated reliability index $\beta=\Phi^{-1}(1-p_f)$, is formally found through the optimization problem:

$$\text{Given } y_{cap}: \quad \beta = \min|\boldsymbol{U}|; \text{ subject to } g(\boldsymbol{U}) = y_{cap} - y(\boldsymbol{U}) = 0 \tag{4}$$

In probabilistic design, however, the capacity y_{cap} is often not given but rather sought, with the goal that a desired reliability β be achieved. This may be solved iteratively with Forward FORM; i.e., assume a capacity y_{cap}, find β through Eq. 4, and iterate with variable y_{cap} until the desired β is found. Alternatively, we may find the capacity y_{cap} which provides a given reliability β, as estimated by Eq. 4, through the "Inverse FORM" method:

$$\text{Given } \beta: \quad y_{cap} = \max y(\boldsymbol{U}); \text{ subject to } |\boldsymbol{U}| = \beta \tag{5}$$

Physically, the Forward FORM method minimizes distance from a known failure surface, finding the most likely failure point in $\boldsymbol{U}$–space. In contrast, in Inverse FORM we specify the exceedance probability p_F and hence minimal distance β to the failure surface. To set the capacity y_{cap} we then search all possible FORM design points with given return period (a hypersphere with radius β), to find the maximum response $y(\boldsymbol{U})$ we must withstand. (We assume throughout that the failure surface is star-shaped with respect to the origin, so that the maximum $y(\boldsymbol{U})$ for $|\boldsymbol{U}| \leq \beta$ occurs along the bounding hypersphere $|\boldsymbol{U}|=\beta$.)

Advantages of Inverse FORM. The formulation of Inverse FORM, Eq. 5, carries several advantages. Perhaps its main benefit lies in decoupling the description of the environmental variables $\boldsymbol{X}$ and the response Y. For desired β the environmental analyst need only report a contour of critical values $\boldsymbol{X}$, corresponding to the sphere $|\boldsymbol{U}|=\beta$ in $\boldsymbol{U}$-space. These contours may then be used to find the specified fractile of any structural response quantity.

Eq. 5 may also confer some computational benefits. It yields y_{cap} without iteration, and it optimizes in one less dimension; e.g., the $n-1$ directions $\phi_i=\cos^{-1}(U_i/\beta)$. Thus, the minimal distance constraint $|\boldsymbol{U}| = \beta$ is used directly to simplify the problem, rather than imposed by trial and error after Forward FORM is done. Also, by recasting the problem in terms of angles ϕ_i, the constraints are simplified into "box-like" regions (e.g., $|\phi_i| \leq \pi$). A greater number of optimization routines are available for this problem, as opposed to those needed in Forward FORM with the nonlinear constraint $g(\boldsymbol{U})=0$. Finally, some experience with the method suggests that it may be better suited than Forward FORM to numerically noisy g-functions.

Example 1: Extreme Wave Crest Heights

Throughout our offshore examples we shall model the wave elevation $\eta(t)$ as a Gaussian process, over a series of stationary seastates. These seastates are then defined by the power spectrum of $\eta(t)$, parametrized here by the significant wave height $H_S=4\sigma_\eta=X_1$, and the peak spectral period $T_P=X_2$. We consider here a Northern North Sea wave climate, for which a Weibull distribution has been fit to H_S (Haver and Nyhus, 1986):

$$P[H_S < h] = F_{H_S}(h) = 1 - \exp[-(h/2.822)^{1.547}] \tag{6}$$

Conditional on H_S, T_P is assumed lognormally distributed with parameters

$$E[\ln T_P|H_S] = 1.59 + 0.42\ln(H_S + 2)\,; \quad Var[\ln T_P|H_S] = .005 + .085\exp(-0.13{H_S}^{1.34}) \tag{7}$$

Following this reference an alternate lognormal distribution is used for H_S values < 3.27 m. This has little impact, however, on the extreme response calculations done here.

Figure 1 shows H_S–T_P contours for response return periods of 10, 100, and 1000 years. These have been found by relating H_S and T_P to standard normal variables U_1 and U_2:

$$H_S = F_{H_S}^{-1}(\Phi(U_1))\,; \quad T_P = F_{T_P|H_S}^{-1}(\Phi(U_2)) \tag{8}$$

From Eq. 5, Eq. 8 gives a contour with return period T_r by varying U_1 and U_2 along the circle $\sqrt{U_1^2+U_2^2}=\beta$, where

$$\beta = \Phi^{-1}(1-p_f) = \Phi^{-1}(1 - \frac{T_{SS}}{365 \times 24 \times T_r}) \tag{9}$$

The factor of 365×24 converts the units of seastate duration T_{SS} [hrs] into those of T_r [yrs]. For example, the T_r=100-year contour follows by setting p_f=.01 per year or 3.43 $\times 10^{-6}$ per 3-hour seastate, so that $\beta=\Phi^{-1}(1-p_f)=4.5$. Note that by including the point $U_1=\beta$, $U_2=0$, this contour contains the wave-dominated seastate from Eq. 2: $H_S=H_{100}$=14.5m and associated median T_P=15.9s. Similarly, the 10- and 1000-year contours in Figure 1 correspond to circles with radius β=4.0 and 5.0, respectively.

The response in this example is the extreme crest height $Y=\eta_{max}$, of interest in setting the deck level to avoid wave impact loads. Y is readily modelled by assuming Poisson upcrossings of level y (e.g., Madsen et al, 1986):

$$P[Y > y] = P[\eta_{max} > y] = \exp\{-(T_{SS}/T_Z)\exp[-8(y/H_S)^2]\} \tag{10}$$

Assuming uncertainty here to be dominated by H_S, we estimate Y by its median value, found by setting Eq. 10 to 0.5:

$$Y = \eta_{max} = y(H_S, T_P) = 0.25 H_S \sqrt{2 \ln(1.44 T_{SS}/T_Z)}; \quad T_Z \approx T_P(1 - 0.29\gamma^{-0.22}) \tag{11}$$

The latter approximation to T_Z is empirical, found to fit fairly well for the JONSWAP wave spectrum with peak factor γ between 1 and 5. Results are shown here for γ=3.3.

Following Eq. 5, the extreme 10-, 100-, and 1000-year wave crests are found by searching the appropriate contour for the maximum response (Eq. 11). Figure 1 shows these extreme crest values to be η_{max}=12.1, 13.7, and 15.2 [m]. Also shown are contours of constant $\eta_{max}(H_S, T_P)$. As might be expected, the (median) extreme wave crest is essentially produced by the largest possible H_S; e.g., the 100-year crest height is produced by the seastate with 100-year H_S (Eq. 2). Note, however, that the same contours remain valid for any structural problem, i.e., for any function $y(H_S, T_P)$, and that other responses may not be dominated by H_S alone. For example, extreme heave motions of tension-leg platforms may be governed by seastates along these contours for which T_P is twice the structural period, due to resonance with second-order load effects (Winterstein et al, 1992).

Note also that these contours are not simply contours of the joint probability density function of H_S and T_P, selected to enclosed area $1 - p_F$. In fact, they will generally enclose somewhat less area. They are instead constructed so that the area inside the failure region, $Y \geq y_{cap}$, is estimated by FORM to be p_F.

Environmental Contours for Stochastic Response

The foregoing analysis assumes deterministic response: in the example, crest height Y is essentially proportional to H_S (Eq. 11). It follows that the seastate with maximum H_S produces the maximum Y. Typically, this assumption is unconservative. For example, it ignores the chance that the largest Y can be produced in a seastate with less-than-maximum H_S. Equivalently, it underestimates Y by neglecting its uncertainty given the seastate parameters; i.e., the difference between the random Y in Eq. 10 and its median estimate in Eq. 11.

To include this uncertainty, we supplement Eq. 3 by adding a random error term ϵ, reflecting conditional uncertainty in Y given $\boldsymbol{U}$:

$$Y = y(\boldsymbol{U}) + \epsilon \tag{12}$$

We define ϵ to have zero median value, so that $y(\boldsymbol{U})$ remains the median response given $\boldsymbol{U}$. In a full analysis, ϵ should be included as an additional random variable. If ϵ is normally distributed, for example, an additional standard normal variable V is introduced into the inverse FORM problem:

$$y_{cap} = \max Y(\boldsymbol{U}, V) = \max y(\boldsymbol{U}) + \sigma_\epsilon V; \text{ subject to } |\boldsymbol{U}|^2 + V^2 = \beta^2 \tag{13}$$

To avoid explicit inclusion of this additional variable, we seek here a new, inflated contour, along which the *median response* $y(\boldsymbol{U})$ yields the correct capacity:

$$y_{cap} = \max Y(\boldsymbol{U}, V = 0) = \max y(\boldsymbol{U}); \text{ subject to } |\boldsymbol{U}| = \beta^* \tag{14}$$

Because this result ignores the conditional uncertainty in Y, to compensate we must choose a contour with larger radius; i.e., $\beta^* \geq \beta$. The value of β^* depends on both σ_ϵ and the precise form of $y(\boldsymbol{U})$. Consider first a simple linear variation of $y(\boldsymbol{U})$ with each U_i:

$$Y(\boldsymbol{U}, V) = m_Y + \sum_i c_i U_i + \sigma_\epsilon V = m_Y + \sigma_Y(\sum_i \alpha_i U_i + \alpha_o V) \tag{15}$$

The latter form is in terms of the total variance of Y, $\sigma_Y^2 = \sum_i c_i^2 + \sigma_\epsilon^2$, and the relative variance contributions $\alpha_i^2 = c_i^2/\sigma_Y^2$ and $\alpha_o^2 = \sigma_\epsilon^2/\sigma_Y^2$ due to U_i and V. Combining Eqs. 13 and 15, the exact Inverse FORM method gives the familiar result

$$y_{cap} = m_Y + \sigma_Y \beta \tag{16}$$

Thus $\beta = (y_{cap} - m_Y)/\sigma_Y$, the ratio of mean safety margin, $E[M]$, to its standard deviation, σ_M. For the linear/normal model of Eq. 15, this gives the exact p_F value. In the reduced Inverse FORM problem (Eqs. 14–15), the exact y_{cap} value requires the inflated contour radius

$$\beta^* = \beta/\sqrt{1-\alpha_o^2} \tag{17}$$

Eqs. 14 and 17 form the basis for "inflated" environmental contours, to compensate for approximating the true stochastic response by its median value. Eq. 17 is best motivated for linear/Gaussian safety margins: replacing a factor by its mean preserves the mean margin $E[M]$, but reduces σ_M^2 to $\sigma_M^2(1-\alpha_o^2)$. (Here, as previously, α_o^2 is the contribution of the omitted variable to σ_M^2.) The reliability index, $\beta = E[M]/\sigma_M$, is then increased by a factor of $1/\sqrt{1-\alpha_o^2}$. Eq. 17 states that it is this artificially inflated β we must seek if we set the omitted variable to its mean value. More generally, if we replace V by an arbitrary value v_o the altered reliability index, β^*, is

$$\beta^*(V = v_o) = (\beta - \alpha_o v_o)/\sqrt{1-\alpha_o^2} \tag{18}$$

This is the FORM omission sensitivity factor (Madsen, 1988). That reference suggests the fixed value $v_o = \beta\alpha_o/2$ so that $\beta^* \approx \beta$. Here we instead retain the median response ($v_o = 0$), and hence inflate the contour through Eq. 17.

In general, $y(\boldsymbol{U})$ will be nonlinear and ϵ non-Gaussian. However, Eq. 15 will apply locally near the design point; indeed it is the basis of the FORM approximation. For relatively small α_o^2, this local linearization may not change significantly after ϵ is omitted, and hence Eq. 17 may remain accurate. This is studied in the examples to follow.

Given β^*, Eq. 9 can be inverted to find an inflated return period T_r^+. Figure 2 shows this inflated return period, for a target return period of $T_r = 100$ years, versus α_o^2. Note that while the reliability ratio β^*/β depends only on α_o^2, the return period ratio T_r^+/T_r depends also on the seastate duration T_{SS} in Eq. 9. Figure 2 shows two cases: (1) all 3-hour seastates are modelled (T_{SS}=3 [hrs]); and (2) only the annual extreme storm is modelled, reflected by taking T_{SS}=1 [yr]=365×24 [hrs] in Eq. 9. For example, to find the 100-year response if α_o^2=.10, we should search the T_r^+=140-year contour of annually occurring seastate parameters, or the T_r^+=320-year contour of 3-hour seastates. If the omitted importance α_o^2 increases to .20, to compensate these return periods must be increased to 215 years (annual seastates) and 1390 years (3-hour seastates).

Of course such results are in a sense circular: the inflated contour radius β^* and return period T_r^+ use α_o^2, whose precise value requires solution of the full FORM or Inverse FORM problem. We hope that growing experience—including results shown here—will suggest a reasonable range for α_o^2. For extreme response of offshore structures we find α_o^2 generally between .05–.25, and most commonly .10–.20. Hence the return period values cited above for α_o^2=.10 and .20 may be useful in estimating likely ranges of response variation.

Example 2: Extremes of Stochastic Wave Crests

To continue our first example, we return to the extreme wave crest problem. Figure 3 shows previous "median extreme crest" results, from the median response in Eq. 11 and the basic contour with return period T_r (Eq. 9). Also shown are "exact" results, which include the actual extreme response distribution

(Eq. 10) and solve the resulting 3-variable FORM problem. As expected these are larger: the actual 100-year extreme crest is found to be 14.9m, as opposed to 13.7m if only the median crest is considered.

We seek here to predict the extreme crest from only a median crest model (Eq. 11), but with an inflated contour. Figure 3 shows that for return periods from 10 to 1000 years, the exact result is bracketed by using inflated contours from Eq. 17, with α_o^2 between .10 and .20. For example, this gives the range 14.5m–15.5m for the 100-year extreme crest, which includes the exact result 14.9m. Figure 3 also shows the result if the inflated contour is used with exact α_o^2, as found from the 3-variable FORM analysis. (Of course this value will not generally be available; however, the comparison serves to test the validity of the theory.) If the exact α_o^2 is used, the inflated contour is found to give rather accurate extreme crest estimates. It is somewhat conservative with respect to the exact FORM result in this case. This reflects that the actual failure surface tends to curve toward the origin, and hence the actual contour should be inflated less than the linear model implies. In principle, curvature (SORM) information could be used to correct for this error.

Example 3: Extreme Base Shear of Shallow-Water Jacket

As a final example, we consider the extreme base shear on a shallow-water jacket structure. From simultaneous hindcast of wind, waves, and current in the Southern North Sea (DHI, 1989), we include six environmental parameters: $\boldsymbol{X}=[H_S, T_Z, U, \Delta D, W, G]$. Here H_S is the annual maximum significant wave height in a T_{SS}=6-hour storm. This storm with annual maximum H_S is also characterized by its mean zero-upcrossing wave period T_Z, current U, surge level ΔD, mean wind speed W and gust factor G.

A correlated model of these variables has been fit to the hindcast data (Haver and Winterstein, 1990; Winterstein and Haver, 1991). These references also establish an analytical estimate of the extreme base shear Y in a seastate, in terms of its corresponding extreme crest height η_{max}:

$$\begin{aligned} Y(\boldsymbol{X}) \quad = \quad & n_{eq}\rho r C_D[\lambda_0 + \lambda_1\eta_{max} + \lambda_2\eta_{max}^2 + \lambda_3\eta_{max}^3] \\ & + C_W C_Z^2 A_P W^2(1+2G) \end{aligned} \tag{19}$$

Here C_D and C_W are drag coefficients for waves and wind, n_{eq} is the equivalent number of legs with radius r for wave loads, and C_Z and A_P are height correction and projected area factors for wind loads. The factors λ_n depend in turn on the wave parameters T_Z, U, and ΔD. Nonlinear dependence of Y on η_{max}, reflected by λ_2 and λ_3, is due to both the nonlinear Morison drag force and the integration of distributed wave forces to the exact water surface.

Figure 4 shows base shear results versus return period, analogous to Figure 3 for the extreme wave crest. The lowest result considers Y a deterministic (median) function of the environmental variables, solving the 6-variable FORM problem by substituting the median crest height (Eq. 11) into Eq. 19. The exact result, larger as expected, follows by solving the 7-variable problem with the full extreme crest distribution (Eq. 10). As in Figure 3, these exact results are bracketed by using the median crest (Eq. 11) with an inflated contour, assuming an omitted importance factor α_o^2 ranging from .10–.20. (In fact, from the exact results we find that α_o^2 varies from .15–.19 in this case.)

Finally, we seek to alter this North Sea example to better reflect Gulf of Mexico storm conditions. First, based on their unimportance in the North Sea case, we set T_Z, ΔD, W and G to their conditional mean values based on H_S. We then revise the two variable (H_S–U) model, preserving the observed correlation ρ=.52 but rescaling the marginal mean current $E[U]$ from 0.4 m/s (North Sea case) to 1.2 m/s. Figure 5 shows resulting H_S–U contours, both including and excluding response variability (in crest height and hence base shear). Excluding this variability, the 100-year H_S–U contour yields the extreme base shear Y=6.90 $\times 10^6$ [N]. If this variability is kept, the exact result Y=7.17 $\times 10^6$ [N] is produced by searching H_S, U, and η_{max} (from its full distribution in Eq. 10). This maximum Y occurs at somewhat smaller H_S and U as shown, but somewhat larger-than-median Y given H_S and U). Our proposed method uses the median Y but searches the inflated contours shown in Figure 5, for which α_o^2=.10 and .20 (and, from Figure 2, T_r^+=140 and 215 years). The resulting Y estimates are shown to be fairly accurate and somewhat conservative, due to failure surface curvature and the slight overestimation in this case of α_o^2 (exact value=.094). Note that in all cases, the design point is at slightly less-than-maximum H_S for the given return period, to accommodate a slightly larger-than-median current given H_S.

Conclusions

- A method has been shown to construct contours of environmental parameters, along which extreme responses with given return period should lie. For any deterministic response these are found by transforming a hypersphere in standard normal space, with radius β (Eq. 9), to the physical space of environmental variables. Environmental contours can thus be produced which, when searched for the maximum response, yield response levels with the desired return periods (Figure 1).
- For stochastic response, the foregoing method will tend to underestimate extreme response levels because it neglects response variability. The degree of error is reflected by α_o^2, the contribution to uncertainty due to the response *given* the load environmental parameters. Across a range of offshore structural problems we typically find α_o^2 values from .05 to .25, and most often from .10 to .20.
- From an assumed α_o^2 value we can determine how to inflate the return period (Figure 2) and hence the environmental contours (Figure 5) along which the median response has the desired return period. The exact result is found often to be well-approximated, and usually bracketed, by choosing contours for which α_o^2=.10 and .20 (e.g., Figures 3–5). Thus, to estimate 100-year levels from only the median response, we should search environmental contours with return periods T_r^+ ranging from about 140–215 years (annual extreme seastates), and about 320–1400 years if all 3-hour seastates are considered.

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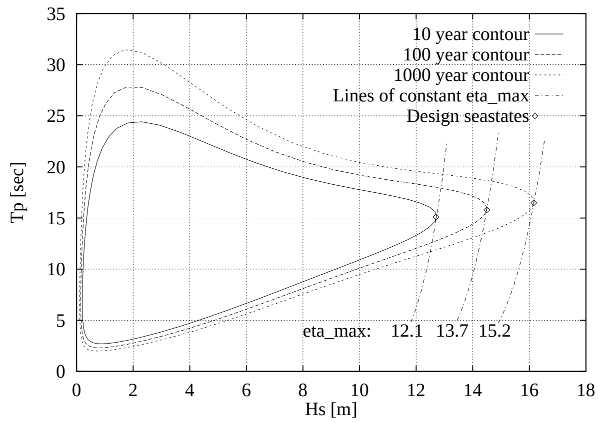

Figure 1: H_S–T_P contours for deterministic response.

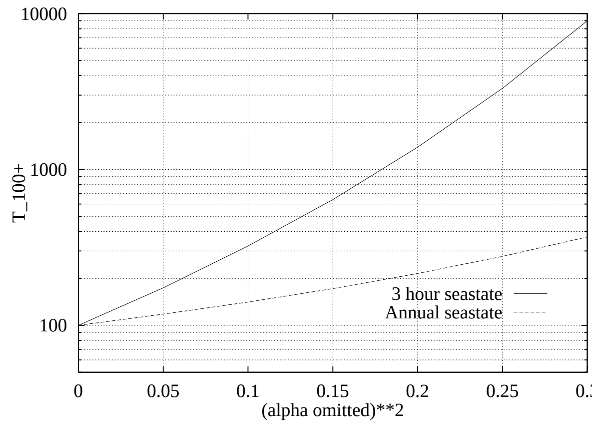

Figure 2: Inflated 100-year return period, T_r^+, for various α_o^2.

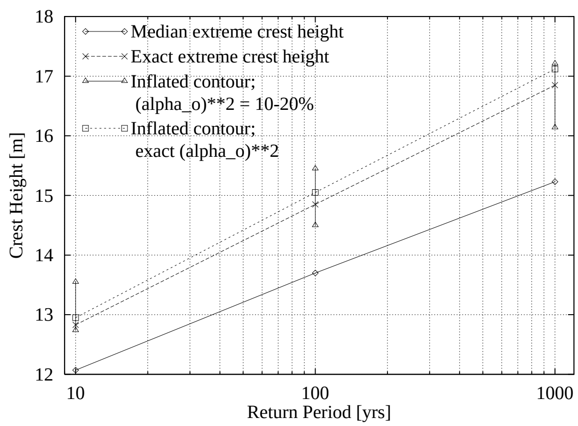

Figure 3: Extreme wave crest with various return periods.

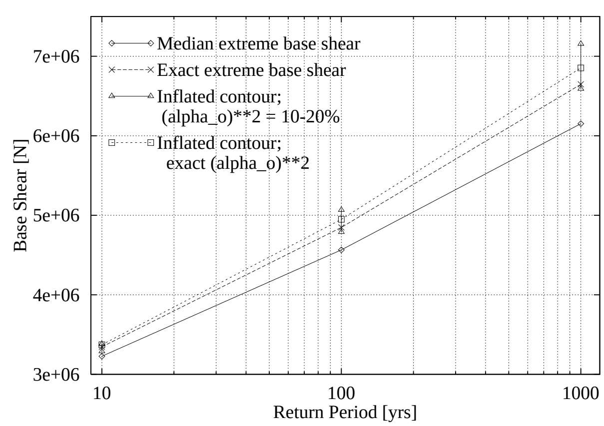

Figure 4: Extreme base shear with various return periods.

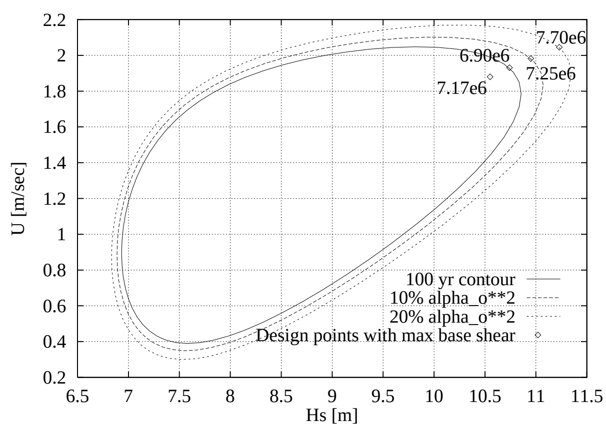

Figure 5: 100-year base shear from various wave height–current contours.