NON-GAUSSIAN AIR GAP RESPONSE MODELS FOR FLOATING STRUCTURES

Bert Sweetman\textsuperscript{1} and Steven R. Winterstein\textsuperscript{2}

Abstract

The air gap response of a specific semi-submersible platform subjected to irregular waves is considered. Statistical analyses are performed on model test data for the absolute near-structure wave elevation, and these measured data are compared with predictions resulting from probabilistic models. Models applied are first-order and second-order diffraction models typical of standard practice, and two new hybrid models that include second-order effects in the incident wave, but not in the diffracted wave. The first of these hybrid models is moment-based, while the second relies on narrow-band random process theory. Either of these new models can be implemented in place of the standard linear-only model with little additional computational effort, as only linear diffraction analysis is required. Both new models are found to better predict the air gap demand than standard linear diffraction analysis.

\textsuperscript{1}\textit{Civil \\& Environmental Engineering Department, Stanford University, Stanford CA 94305-4020, USA, bert.sweetman@stanford.edu}

\textsuperscript{2}10160 Parkwood Drive \#4, Cupertino CA 95014, USA
Introduction and Background

Design practice is generally more well-established for fixed platforms than for their floating counterparts, so it is reasonable to consider these well established practices for design of floating platforms. Existing practice for calculating design air gap levels for fixed platforms employs a design wave approach based on the 100-year extreme wave crest level. Jacket structures, which predominate in many geographical areas, are slender relative to the length of the incident waves and so experience only small diffraction effects. Thus, application of the design wave approach is reasonable to the extent that the global structural behavior is quasi-static.

Higher volume structures, whether fixed or floating, complicate the air gap calculation by significantly diffracting the incident waves. For these structures ignoring diffraction effects is non-conservative in that diffraction effects generally worsen the air gap demand. Large-volume floating structures, including semi-submersibles and floating production, storage and offloading vessels (FPSO’s), offer the most significant challenge. Two distinct hydrodynamic effects are observed: (1) global forces and resulting motions are significantly affected by diffraction; and (2) the local wave elevation, \( \eta(t) \), is also significantly influenced by diffraction. For semi-submersibles, these wave amplification effects are most extreme at locations above a pontoon and/or near a major column.

Present air gap design methodologies for floating structures are not standard and rely heavily on empirical knowledge and model tests. Most of the experience with semi-submersibles has been in support of the drilling industry. Large production semi-submersibles, novel large-volume offshore structures, and the Mobile Offshore Base (MOB) provide design challenges that are well outside the present design experience base. These vessels are significantly larger than drilling semisubmersible platforms and, unlike drilling semisubmersibles, are generally required to remain on station throughout the most severe weather. Urgency is added by the fact that air gap design problems (wave impacts) have been encountered on large North Sea semi-submersibles, including the Veslefrikk B platform in the Norwegian sector of the North Sea.

Airgap modeling and prediction has been the subject of previous work performed at Stanford University. A fractile-based approach has been applied to develop a scaling factor between the statistics of the incident waves and those of the associated airgap demand (Winterstein & Sweetman, 2001). Application of this
fractile-based approach considers the adequacy of first-order incident wave effects and first-order diffraction theory. In other work, comparisons have been made between airgap predictions resulting from various combinations of first- and second-order incident wave effects and first- and second-order diffraction theory with physical model test results (Manuel et al., 2001). The present work lies between these two previous works: here, first- and second-order incident wave effects are considered but the results of only first-order diffraction theory are used to predict the airgap response.

Theoretical Overview and Motivation

The disturbed water surface in the presence of the vessel, $\eta(t)$, is assumed to be a sum of incident and diffracted waves, $\eta_i$ and $\eta_d$, each of which is a sum of first- and second-order components:

$$\eta(t) = \eta_i(t) + \eta_d(t)$$  \hspace{1cm} (1)

$$\eta_i(t) = \eta_{i,1}(t) + \eta_{i,2}(t)$$  \hspace{1cm} (2)

$$\eta_d(t) = \eta_{d,1}(t) + \eta_{d,2}(t)$$  \hspace{1cm} (3)

This assumption is consistent with most state-of-the-art nonlinear hydrodynamic analyses, which employ second-order perturbation solutions. Combining Equations 1–3:

$$\eta(t) = \eta_{i,1}(t) + \eta_{i,2}(t) + \eta_{d,1}(t) + \eta_{d,2}(t)$$  \hspace{1cm} (4)

In a conventional second-order analysis, all four terms in the sum are considered; in a first-order analysis, only the first two terms are considered. In the two new “hybrid” methods proposed here, second-order effects of the incident waves are considered, while second-order diffraction effects are neglected. Thus, the new methods proposed here lie between first- and second-order analysis in that the first three terms in the sum are considered.

Two new methods of applying first-order diffraction results are introduced in this paper. The difference between these two methods is in the application of the Stokes second-order wave component. In the method denoted “Stokes second-order,” a square matrix of Stokes second-order transfer function components is developed and applied in Equations 13–16 (to be discussed). In the method based on narrow-band theory,
the extreme of the total wave process, \( \eta(t) \), is assumed to coincide in time with that of \( \eta_1(t) \). It is therefore only necessary to model second-order effects during that single, largest wave cycle, so the Stokes second-order contribution is calculated for only that largest wave. Special consideration of only the single largest wave cycle is conceptually similar to the design wave approach. The advantage of either of the “hybrid” approaches is computational simplicity: \( \eta_{1,s} \) requires only (relatively straightforward) linear diffraction analysis, and \( \eta_{2,s} \), which is available analytically from second-order Stokes theory.

Second-order diffraction analyses are available (e.g., WAMIT 4.0, 1995) but these second-order methods are difficult to apply and presently lack widespread use and verification in modeling the nonlinear diffracted wave surface. Standard airgap response prediction uses linear theory, which generally does not effectively reproduce measurements from model tests (Winterstein & Sweetman, 2001, Teigen & Trulsen, 2001, Manuel et al., 2001 and Manuel & Winterstein, 2000). First-order diffraction is often selected over the more powerful second-order because of its relative simplicity, ease of use and robustness of the solution. While second-order diffraction effects are expected to better reflect observed data, these radiation/diffraction panel calculations have been found sometimes to over-predict airgap demand (Sweetman et al., 2001b and Teigen & Trulsen, 2001).

Figure 1 shows the results from conventional application of first- and second-order theory. First-order theory results in a relatively slight under-prediction of the mean maxima, while second-order theory results in a dramatic over-prediction. Second-order system identification has been applied to measured airgap data (Sweetman et al., 2001a) from which it has been determined that the over-prediction results from high-frequency second-order terms. An improved second-order method has been proposed (Sweetman et al., 2001b) in which these seemingly aberrant second-order terms are replaced by those from Stokes theory. The results of this improved second-order method are shown in Figure 2 denoted “‘Best’ Application of Second-Order Diffraction.” The results denoted “Narrow Band Model” and “Stokes Second-Order” are the principle subject of this paper.

Neglecting second-order diffraction substantially simplifies computations but is clearly expected to underestimate near-column diffraction effects; such an underestimation has been shown for this platform (Winterstein & Sweetman, 2001). The results in Figure 2, however, show that optimal use of second-order
diffraction results also underestimates these effects. A goal here is to understand the numerical impact of this simplification through use of two different stochastic models.

**Air Gap Notation and Modeling Issues**

Figure 3 shows a schematic view of a semi-submersible platform, both before and after waves are applied. In the absence of waves, the still-water airgap distance is denoted $a_0$. In the presence of waves, $\eta(t)$ denotes the wave surface elevation relative to a vertically fixed observer at a particular horizontal location along the moving structure. The corresponding vertical motion of the platform is denoted $\delta(t)$. If $\eta = \delta$, the airgap would remain equal to its still-water value, $a_0$. More generally, the airgap response $a(t)$ will be reduced from $a_0$ by the difference, $\eta(t) - \delta(t)$:

$$a(t) = a_0 - [\eta(t) - \delta(t)]$$

(5)

Deck impact occurs if the airgap $a(t) < 0$. Among the various terms in Equation 5, the vertical offset $\delta(t)$ is perhaps the most straightforward to model. Linear diffraction results may often suffice to accurately model this offset. In contrast, the free surface elevation, $\eta(t)$, generally shows nonlinear behavior—and hence represents a non-Gaussian process. Our modeling attention is therefore focused here on $\eta(t)$. This separation of vessel motions from diffraction effects on the water surface is fairly standard in hydrodynamic post-processing, and is consistent with typical hydrodynamic diffraction analysis.

**Description of Model Test Data**

Test data considered here come from a 1:45 length-scale model of Veslefrikk, which was tested in the wave tank at Marintek using various types of irregular waves (Fokk, 1995). Figure 4 shows a plan view of the platform, together with the 9 locations for which the air gap responses have been measured as a function of time. Note that airgap probes with lower numbers are generally further up-stream, i.e., closer to the wave generator. All tests studied here apply long-crested waves traveling along the diagonal of the structure.

The platform rigid-body motions in heave, roll, and pitch—denoted $\xi_3$, $\xi_4$, and $\xi_5$—have also been
recorded. This permits estimation of the net vertical displacement, $\delta(t)$, at any location $(x, y)$ of interest:

$$
\delta(t) = \xi_3(t) + y \cdot \sin(\xi_4(t)) - x \cdot \sin(\xi_5(t))
$$

Table 1 summarizes the geometric properties of the platform as configured for the tests used here. Prior to the model test, waves are first generated in the model test basin in absence of the model. The incident wave, $\eta_i(t)$, is measured at location 7 (Figure 4), where the platform is to be centered. Following common practice, wave histories have been generated from a stationary random process model, applied over a fixed “seastate” duration of $T_{ss}=3$ hours. Its spectral density function, $S_\eta(f)$, is described by the significant wave height $H_S=4\sigma_\eta$, the peak spectral period, $T_P$, and the spectral peakedness factor $\gamma$. Table 2 describes the $H_S$, $T_P$, and $\gamma$ values for each of the three test conditions. The $\gamma$ reported for the bimodal spectrum is an equivalent steepness.

The time-history of the elevation of the water-surface relative to a fixed observer is inferred using Equation 5 and the measured airgap, $a(t)$, and the estimate of $\delta(t)$ as:

$$
\eta(t) = a_0 - a(t) + \delta(t)
$$

The first four statistical moments: mean, $m_\eta$, standard deviation, $\sigma_\eta$, skewness, $\alpha_{3\eta}$, kurtosis, $\alpha_{4\eta}$, and Peak Factor, PF, are calculated from this time-history as:

$$
m_\eta = E[\eta(t)]
$$

$$
\sigma^2_\eta = E[(\eta(t) - m_\eta)^2]
$$

$$
\alpha_{3\eta} = \frac{1}{\sigma^3_\eta}E[(\eta(t) - m_\eta)^3]
$$

$$
\alpha_{4\eta} = \frac{1}{\sigma^4_\eta}E[(\eta(t) - m_\eta)^4]
$$

$$
PF = \frac{\eta_{max} - m_\eta}{\sigma_\eta}
$$

Here the “E” notation signifies the expectation (or averaging) operation. These “measured” statistical moments are used for comparison with theoretical predictions in Figures 5 and 6.

The “Measured” statistical moments and peak factors are calculated applying Equations 8–12 to each of the 3-hour realizations of a target seastate. The resulting values are averaged to determine the “average
of measured” statistics for each seastate. Error bars are calculated as the standard deviation of the \( n \) observations divided by the \( \sqrt{n} \), where \( n \) equals 5 for the 12 meter seastate and 6 for the 14 meter seastate.

**Theoretical Moment-Based Models and Results**

A common theoretical basis is applied to develop the results presented in Figure 1 denoted “Full Second-Order” and those presented in Figure 2 and those denoted “Stokes Second-Order” in Figures 5–6. The difference between the methods lies in the inclusion of second-order diffraction results. Whether second-order diffraction effects are included or neglected, the first- and second-order processes \( \eta_1(t) \) and \( \eta_2(t) \) can be rewritten in terms of standard Gaussian processes, \( u_j(t) \):

\[
\eta_1(t) = \sum_{j=1}^{2n} c_j u_j(t) ; \quad \eta_2(t) = \sum_{j=1}^{2n} \lambda_j u_j^2(t)
\]

in which \( n \) is the number of frequency components of \( \eta_1(t) \). For a full second-order calculation, \( \eta_2(t) = \eta_{2,i}(t) + \eta_{2,d}(t) \); for a Stokes second-order calculation \( \eta_2(t) = \eta_{2,i}(t) \). In either case, the coefficients \( c_j \) and \( \lambda_j \) can be obtained by solving an eigenvalue problem of size \( n \) for problems involving difference or sum frequencies only (Naess, 1986, Naess, 1992), or of size \( 2n \) for problems—such as these airgap responses—which involve both (e.g., Winterstein et al., 1994a). The resulting moments of \( \eta \) can be found directly from the coefficients \( c_j \) and \( \lambda_j \):

\[
\sigma_{\eta}^2 = \sum_{j=1}^{2n} (c_j^2 + \lambda_j^2)
\]

\[
\alpha_{3,\eta} = \frac{1}{\sigma_{\eta}^2} \sum_{j=1}^{2n} (6c_j^2 \lambda_j + 8\lambda_j^3)
\]

\[
\alpha_{4,\eta} = 3 + \frac{1}{\sigma_{\eta}^2} \sum_{j=1}^{2n} (48c_j^2 \lambda_j^2 + 48\lambda_j^4)
\]

Figures 5 and 6 are used to compare the statistical moments and peak factors observed for \( \eta(t) \) with those predicted from Equations 14–16 with second-order contributions from only the incident waves. Comparing first the standard deviation values from theory and measurement, reasonably good agreement is found between the first- and second-order results in both magnitude and behavior across the 9 field-points. Recalling that all of these results neglect second-order diffraction effects, it appears that first-order theory may be sufficient to accurately predict the rms level of the wave elevation, and hence that of the air gap response.
Figures 5 and 6 show similar comparisons for the third and fourth statistical moments. Recall that the only source of non-Gaussian behavior in the Stokes second-order model is that of the incident wave; it is therefore not surprising that these theoretical moments are relatively constant across field points. The best result that could be reasonably expected is that the theoretical higher moments roughly equal those of the incident wave. In fact, the second-order theory appears to accurately predict the wave skewness, but under-estimates its kurtosis (roughly 3.1 vs 3.3). This finding is consistent with a detailed Ph.D. study of second-order random waves (Jha, 1997; Jha & Winterstein, 2000). Beyond the theoretical underestimation of the “background” kurtosis level of the incident wave, the most striking feature of these figures is the strongly enhanced nonlinear behavior (increased skewness and kurtosis) observed at near-column locations (e.g., locations 1 and 5). It is clear that this nonlinear location-specific behavior cannot be modeled through linear diffraction alone.

The plots of peak factors in Figures 5 and 6 show the resulting moment-based predictions of the peak factor. The peak factor is a dimensionless indicator of the extremes of the process.

The “First-Order” results are based on first-order diffraction results and a Gaussian random process model. The expected maximum of a standard Gaussian process, $U(t) = \eta(t)/\sigma_\eta$, in $N$ cycles is estimated as (e.g., Madsen et al., 1986 from Cartwright & Longuet-Higgins, 1956 and Davenport, 1964):

$$E[U_{\text{max}}] = \frac{E[\eta_{\text{max}}]}{\sigma_\eta} = \sqrt{2 \ln N} + \frac{0.577}{\sqrt{2 \ln N}}$$

Equation 17 gives an estimate of the average peak factor, $PF$, comparable to that defined by Equation 12. Application of Equation 17 to a 3-hour seastate with a typical $N \approx 1000$ results in the peak factor, $PF \approx 3.9$.

The larger peak factors in the figures reflect non-Gaussian behavior. Equation 17 consistently under estimates the peak factor, yielding predictions less than 4.0 compared with measurements of 4.5 to 5.5.

Estimated extremes for non-Gaussian processes (here, the Stokes second-order and full second-order) are estimated using the Hermite model and theoretical skewness and kurtosis estimates from Equations 14–16. The Hermite model assumes the non-Gaussian process $\eta(t)$ to be a cubic transformation of a standard Gaussian process $u(t)$, conveniently rewritten as a sum of Hermite polynomials (Winterstein, 1988):

$$\eta = g(u) = m_{\text{eta}} + \kappa \sigma_x [u + c_{3H}(u^2 - 1) + c_{4H}(u^3 - 3u)]$$

(18)
The variance of \( \eta \) is preserved by setting \( \kappa = [1 + 2c_3^2 + 6c_4^2]^{-1/2} \). The coefficients \( c_3H \) and \( c_4H \) are determined to preserve the desired skewness and kurtosis. (Results presented in this paper are calculated using a numerical routine to “optimize” \( c_3H \) and \( c_4H \), to minimize error in matching skewness and kurtosis values; e.g., Winterstein et al., 1994b). Assuming the same transformation \( g \) in Equation 18 applies at every point in time, including those points which are the maxima:

\[
\eta(t) = g(u(t))
\]

\[
\eta_{\text{max},T,p} = g(u_{\text{max},T,p})
\]

Here, \( \eta_{\text{max},T,p} \) refers to the \( p^{th} \) fractile of the maximum water surface elevation, \( \eta_{\text{max},T} \), observed in time period, \( T \). Equation 20 gives a direct relation between \( \eta_{\text{max},T,p} \) and \( u_{\text{max},i,p} \), which is the corresponding Gaussian extreme fractile of the non-Gaussian water surface elevation. Consistent with Type I extreme value theory, the mean value of \( \eta_{\text{max},T} \) is assumed to correspond to its 57\% fractile.

\[
E[\eta_{\text{max},T,p}] = \eta_{\text{max},T,0.57} = g(u_{\text{max},T,0.57})
\]

Conceptually restating the implications of Equation 21: \( E[\eta_{\text{max}}] \) is the expected value (average) of a hypothetical collection of observed 3-hour maxima. Each of these 3-hour maxima is the maximum observed value of \( \eta \) in a 3-hour observation. Use of the 57\% fractile is consistent with Type I extreme value theory, e.g. the mean maximum, \( \eta_{\text{max},T,0.57} \), is exact if the collection of observed peaks follows a Gumbel distribution.

Equations 17 and 21 serve as the basis of the “Stokes Second-Order” results in the peak factor plot in Figures 5 and 6. In this application, the mean of the process, \( m_\eta \), is set equal to zero. As might be expected, the peak factor estimates from second-order theory are always superior to the Gaussian results in the peak factor plot in Figures 5 and 6. Nonetheless, these second-order predictions fail to adequately follow the trend in observed extremes at near-column locations. This inadequacy at near-column locations may also be anticipated: the prediction model does not include nonlinear diffraction effects caused by the presence of the structure. This pattern of deviation, between measured peak factors and second-order predictions, closely follows the pattern of deviation in skewness and kurtosis from the preceding figures.
Narrow-Band Model

Results to this point can be summarized as follows: the first three plots in each of Figures 5 and 6 show that use of first-order diffraction results underestimates the coefficients of skewness \( \alpha_3 \) and kurtosis \( \alpha_4 \). First-order diffraction cannot predict the enhanced nonlinear effects (larger \( \alpha_3, \alpha_4 \)) at near-column locations. As a result, using these moment estimates leads to a similar underestimation of extreme wave levels (the fourth plot, peak factors, in each figure).

In contrast, Figures 5 and 6 show that the standard deviation of the spatially varying wave surface is relatively accurately predicted with only first-order diffraction effects. Since \( \eta(t) \) is a zero-mean Gaussian process, knowledge of its standard deviation generally suffices to estimate its mean extreme (as in Equation 17):

\[
E[\eta_{1, \text{max}}] = \sigma_\eta \sqrt{2 \ln N} + \frac{0.577}{\sqrt{2 \ln N}}
\]

If the extreme of the total wave process, \( \eta(t) \), is assumed to coincide in time with that of \( \eta_1(t) \), it is only necessary to model second-order effects during that single, largest wave cycle. Performing load calculations for only the single most extreme event is the concept underlying the design wave approach. Modeling of second-order effects is accomplished using a narrow-band random process model.

Longuet-Higgins (1952) theoretically showed the peaks of a narrow-band Gaussian wave time-history follow a Rayleigh distribution. Other developments regarding application of narrow-band theory to ocean waves have been undertaken by Tayfun (1980, 1986a, 1986b and 1989). Applying a standard narrow-band model to the Gaussian process \( \eta_1(t) \),

\[
\eta_1(t) = a(t) \cos[\omega t + \theta_1(t)]
\]

in terms of the slowly varying amplitude \( a(t) \) and phase \( \theta_1(t) \). (The instantaneous frequency \( \omega \) may also be considered to be slowly varying.) The resulting second-order correction is then

\[
\eta_2(t) = a^2(t)H_2^+(\omega, \omega) \cos(2[\omega t + \theta_2(t)])
\]

in which \( H_2^+ \) is the second-order sum frequency transfer function. (The difference frequency transfer function, \( H_2^- \), for a sinusoidal input gives only a constant offset, which does not contribute to oscillations about the mean wave surface.)
An important byproduct of the narrow-banded assumption affects the outcome of Equations 23-24: the narrow-band model effectively phase-locks the first- and second-order components, i.e $\theta_1(t) = \theta_2(t)$. In the largest wave cycle each process is assumed to attain a peak value at the same time (when both cosine terms are unity). At this time $a(t) = \eta_{1,\text{max}}$, so that the mean extreme of the total wave $\eta = \eta_1 + \eta_2$ is estimated as

$$E[\eta_{\text{max}}] = E[\eta_{1,\text{max}}] + E[\eta_{1,\text{max}}^2]H_2^+(\omega, \omega) \approx E[\eta_{1,\text{max}}] + E[\eta_{1,\text{max}}^2]\frac{k}{2}$$

(25)

The expression $H_2^+ = k/2$, in which $k$ is the wave number, follows from the second-order Stokes model in infinite water depth. The wave number $k = \omega^2 / g$, in which $g$ is the acceleration due to gravity, should be based on a wave frequency “characteristic” of extreme waves. The characteristic wave frequency, $\omega_c$, is estimated here as $\omega_c = 2\pi / (0.92T_{1/2})$, although results are expected to be relatively insensitive to this precise definition. Finally, Equation 22 is used to evaluate $E[\eta_{1,\text{max}}]$, which is combined with $\sigma_{\eta_{1,\text{max}}}^2 = (\pi^2/6)(\sigma_{\eta_{1}}^2 / (2 \ln N))$ (e.g., Madsen et al., 1986) to estimate $E[\eta_{1,\text{max}}^2]$:

$$E[\eta_{1,\text{max}}^2] = E[\eta_{1,\text{max}}]^2 + \sigma_{\eta_{1,\text{max}}}^2 \approx \sigma_{\eta_{1}}^2 \left\{1 + \frac{0.577}{\sqrt{2 \ln N}} + \frac{\pi^2}{6 \cdot 2 \ln N} \right\}$$

(26)

Comparison of All Models and Data

Figure 7 is used to compare the observed mean extreme wave elevation with the predictions from all three preceding models. These predictions come from the conventional linear, moment-based nonlinear, and the narrow-band nonlinear models (given respectively by Equations 17, 21, and 25). Results for the first two models have already been given, in terms of peak factors, in Figures 5 and 6. Figure 7 shows all results in terms of the mean extreme level. Recall that all three methods rely on the same linear diffraction analysis results, and rely on only these diffraction results.

As expected from previous results, the standard linear model is generally non-conservative, under-predicting the airgap demand for all wave probe locations and for both sea-states. The air gap demand extremes are typically under-predicted (compared with “Observed”) by at least 20%, and sometimes substantially more, particularly at near-column locations. Recall from the plots in Figures 5 and 6 that the
standard deviation of this airgap process is typically under-predicted by 10% or less.

Both the moment-based and narrow-band models predict higher airgap demands than the standard linear model, which consistently under-predicts the airgap response. The under-prediction of $\alpha_3$ and $\alpha_4$ in the moment-based model (the skewness and kurtosis plots in Figures 5 and 6) propagate through the analysis, so that the extremes of the process are also somewhat under-predicted. The spatial variability of $\eta_{max}$ results entirely from spatial variability of the first-order transfer functions, which is captured in the standard deviations of the processes as presented in Figures 5 and 6.

Extreme estimates from the narrow-band model are found to be more conservative than either the first-order or Stokes second-order model. The higher predictions resulting from the narrow-band model is as expected: the assumed spectral narrowness of the first-order process causes complete phase-locking between the first- and second-order peaks. The air gap demand is very well predicted at location 5, which has proven very hard to predict, and where wave impact was found in the field and repeated in this set of model tests. The apparent success at this location, however, may in part be due to offsetting effects: the standard deviations of the processes are over-predicted at this location (Figures 5 and 6) and the second-order diffraction effects are neglected (Equation 25).

Conclusions

Several methods of estimating air gap demand have been compared with detailed model tests of the Veslefrikk semi-submersible platform. A standard linear diffraction model typical of common practice has been shown to under-predict the airgap demand for all wave probe locations. A standard second-order diffraction model as been shown to dramatically over-predict this demand. Air gap demand extremes are typically under-predicted by the first-order model by 20% or more, while the airgap demand standard deviation is typically under-predicted by 10% or less.

Two new models are proposed that include second-order effects in the incident waves, but that do not require the relatively difficult second-order diffraction calculations. The first model is moment-based and the second relies on narrow-band random process theory. Either model can be implemented in place of the more standard linear-only model with little additional computational effort because only linear diffraction
analysis is required. Each model better predicts the airgap demand than standard linear diffraction analysis.

The narrow-band model is derived by neglecting second-order diffraction effects during only the largest wave cycle and by assuming the first and second-order parts of the wave cycle are phase locked. A relatively simple closed-form expression is found for direct estimation of extremes (Equation 26). Resulting extreme estimates are found to be more conservative and to better predict spatial variability than those resulting from either standard linear theory or the moment-based model (Figure 7).

The moment-based model requires an eigenvalue analysis to determine the statistical moments of the resulting non-Gaussian process. The skewness and kurtosis, $\alpha_3$ and $\alpha_4$, are under-predicted in absence of second-order diffraction effects. The extremes of the process are conveniently predicted from these moments using Hermite models. Despite the under prediction and insufficient spatial variability of the extremes associated with underestimation of $\alpha_3$ and $\alpha_4$ (Figures 5 and 6), this model better predicts the air gap demand than standard linear theory.

Acknowledgements

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References


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Table 1: Characteristics of Veslefrikk platform.
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<td>13.5</td>
<td>3.0</td>
<td>6</td>
<td>JONSWAP</td>
</tr>
</tbody>
</table>

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