FATIGUE OF RISERS: CALIBRATING RELIABILITY ESTIMATES FROM FULL-SCALE DATA

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ABSTRACT

A number of measurement campaigns have sought to quantify fatigue damage on drilling and production risers in the field. The data from these campaigns have recently been used to test the accuracy of software predictions of riser motions and hence fatigue. It is often found that state-of-the-art software shows a conservative bias in predicting fatigue damage—conservative factors of 10 or more. At the same time, short-term damage estimates (e.g., over 10-minute intervals) show significant scatter: COV (coefficient of variation) values of 1.0–2.0 are typical.

We suggest methods here to better incorporate the information from these measurements into fatigue reliability predictions. We note that (1) software error statistics show markedly different behavior in different regimes; e.g., for different levels of predicted damage; and (2) our fatigue reliability concern lies with the variability not in individual 10-minute damage rates, but rather in the long-run damage rate over the entire structural life. These features suggest a non-parametric approach, in which software error statistics are binned and separately analyzed for different predicted damage levels. The sample means in each bin, together with their variabilities, then form the basis for our reliability assessment.

Because our interest lies with the average behavior in each bin, variability reduces notably: COVs of 1.0–2.0 (on individual 10-minute rates) are reduced by an order of magnitude or more. This variability is commonly dominated by the uncertainty in fatigue capacity (S–N curve, Miner’s rule, etc.) Thus, our measurement campaigns are of most interest in predicting only the mean bias in damage predictions; conventional methods are then available to propagate these into a standard fatigue reliability analysis. Results are shown for a number of applications, and implications considered for design (i.e., load factors).

INTRODUCTION

Risers on offshore structures are commonly subject to fatigue. Their typical design basis uses analytical models of riser motions/stresses, together with Miner’s rule, to predict fatigue life. We then require that this predicted life exceed the actual service life by a prescribed safety factor—commonly a factor of 10. Note that this factor of 10 does not assume that the software producing the prediction is conservative (in damage terms) by a factor of 10. It typically assumes the software is unbiased, and the factor of 10 is intended to cover the rather large uncertainty in fatigue capacity (number of cycles to fail under given loading).

Recently, a number of measurement campaigns have been used to test the accuracy of software in predicting riser motions. This is of special interest in the technically challenging area of VIV (vortex-induced vibrations). In these cases, conventional software such as SHEAR7 [1] often yields strongly conservative estimates of fatigue damage. At the same time, the biases in short-term damage estimates (e.g., over 10-minute intervals) show significant scatter: COV (coefficient of variation) values of 1.0–2.0 are typical. The question, which has been pursued over a number of years in the technical literature [2–9], is how to incorporate the information of these measurements to better assess riser fatigue. This question, of course, is of interest for WIF (wave-induced fatigue) as well.
This paper seeks to improve the methodology to address both VIV and WIF problems. Notable features of these problems include:

1. Software error statistics show markedly different behavior in different regimes; e.g., for different levels of predicted damage.
2. Our fatigue reliability concern lies not with the variability in individual 10-minute damage rates, but rather in the long-run damage rate over the entire structural life.

Both of these features suggest a non-parametric approach, in which software error statistics are binned and separately analyzed for different predicted damage levels. The sample means in each bin, together with their variability, then form the basis for our fatigue reliability prediction.

Because our interest lies with the average behavior (in each bin), variability reduces notably: COVs of 1.0–2.0 (on individual 10-minute rates) are reduced by an order of magnitude or more. This variability is commonly dominated by the uncertainty in fatigue capacity (S–N curve, Miner’s rule, etc.) Thus, our measurement campaigns are of most interest in predicting the mean bias in damage predictions; conventional methods are then available to propagate these into a standard fatigue reliability analysis. Results are shown for various applications, and implications considered for design (i.e., load factors).

APPLICATIONS TO RISER VIV
VIV of a Gulf of Mexico Drilling Riser

We first consider a MODU drilling riser subjected to highly sheared currents, typical in the Gulf of Mexico. We use an extensive dataset, comprised of \( n = 363 \) 10-minute VIV episodes. During each episode, measured riser motions are used to infer the fatigue damage rate, \( \dot{D}_i \), based on (1) a standing wave response model and (2) Miner’s rule. For each episode, a corresponding predicted damage rate, \( \dot{D}_{pred,i} \), is found from the software code SHEAR7. The maxima along the length, for each case, are then compared. This dataset has been studied extensively; for example, Refs [3,9] have compared measurements with version 4.4 of SHEAR7. Here we instead use version 4.6 of this software, with the developer-recommended parameters for bare risers.

Figure 1 compares the measured and predicted damage rates. It is clear that the predictions span an extremely wide range—more than 7 orders of magnitude, from \( \dot{D}_{pred} = 9.8 \times 10^{-7} \) to 2.17 [per year]. It is also clear that the predictions often fail to closely follow the measurements. In particular, large predicted values tend to be conservative, while low predictions are often non-conservative. Indeed, the data show two distinct “clouds,” of non-conservative and conservative behavior respectively, separated at around the \( D_{pred} = 10^{-4} \) level. (There are considerably more data in the rightmost, conservative cloud.)

These observations motivate our statistical study of the modeling error

\[
X_i = \frac{\dot{D}_i}{\dot{D}_{pred,i}}
\]  

(1)

in which \( \dot{D}_i \) is the damage rate inferred from the measurements, and \( \dot{D}_{pred,i} \) is the damage rate predicted by the software. In particular, we separate the data into different bins of \( \dot{D}_{pred,i} \), and
estimate the mean and standard deviation of $X_i$ in each bin:

$$m_{X_i} = E[X_i | D_{pred,i}]$$

$$\sigma_{X_i} = \sqrt{\text{Var}[X_i | D_{pred,i}]}$$

Figure 2 shows the result1. Note that conservative predictions are here associated with $X < 1$. The figure thus shows a clear trend toward increasing conservatism (decreasing mean $X$) with increasing $D_{pred}$. The standard deviation $\sigma_X$ tracks rather closely with its mean, suggesting a coefficient of variation, $\sigma_X/m_X$, on the order of 1.0. This is consistent with previous findings. (Note that one bin contains only a single 10-minute data point, hence a standard deviation cannot be estimated.)

Modelling Issue 1: In previous work (e.g., [3, 9]), the modelling errors $X_i$ were effectively taken as independent of $D_{pred,i}$. This ignores the strong conservative trend shown in Fig. 2 and similar datasets.

Long-Term Reliability Analysis

We assume that $D(T)$, the value of Miner’s damage after a sufficiently large number of cycles, is well-approximated by its long-run average:

$$D(T) = D_{LT} T$$

This long-run damage rate, $D_{LT}$, can in turn be decomposed into a weighted mixture of damage rates that occur in different current conditions:

$$D_{LT} = \sum_i p_i D_{pred,i} m_{X_i} = \sum_i d_i m_{X_i}; \quad d_i = p_i D_{pred,i}$$

Here $D_{pred,i}$ is the damage rate predicted in current conditions $i$, and $p_i$ is the long-run fraction of time this current condition acts. The result depends only on their product, $d_i = p_i D_{pred,i}$, which can be viewed as a “damage density.” The sum $\sum_i d_i$ yields the long-run damage estimated by the software. The mean correction factors $m_{X_i}$ (Eqn. 2) are introduced to account for software biases.

Having binned data based on $D_{pred,i}$, $m_{X_i}$ values are estimated by $\bar{X}_i$, the sample mean observed in that bin. These are the mean values shown in Fig. 2. The statistical uncertainty in our damage rate estimate (Eqn. 5) can then be expressed by the variance

$$\text{Var}[D_{LT}] = \sum_i \sum_j d_i d_j p_{ij} \frac{\sigma_X \sigma_{X_j}}{\sqrt{n_in_j}}$$

Here $n_i$ and $n_j$ are the number of data in bins $i$ and $j$, $\sigma_{X_i}$ and $\sigma_{X_j}$ are the conditional standard deviations (Eqn. 3), and $p_{ij}$ is the correlation coefficient between $X_i$ and $X_j$.

Modelling Issue 2: In previous work (e.g., [3, 9]), the variance of $X$ was taken as the variability shown in the raw data; i.e., the 10-minute damage rate values. The $\sqrt{\text{Var}}$ term in Eqn. 6 reflects our interest here in the variability of the long-run damage rate, not of individual 10-minute damage rates.

In the simplest case, the failure time $T_f$ is taken as $1/D_{LT}$, the time when Miner’s damage $D(T_f)=1$. In probabilistic analysis, it is common to generalize this by predicting failure when $D(T_f)$ exceeds a (random) threshold $\Delta$. It follows that

$$T_f = \frac{\Delta}{D_{LT}}$$

Corresponding moments of $T_f$ can be estimated as

$$E[T_f] \approx \frac{E[\Delta]}{D_{LT}}; \quad V_{T_f} \approx \sqrt{\frac{\sigma^2}{\Delta^2} + \frac{\sigma^2_{\Delta}}{D_{LT}^2}}$$

In the second of these results, $V$ refers to the coefficient of variation (ratio of standard deviation to mean) of the subscripted variable. (This result becomes exact if each $V$ is replaced by the standard deviation of the logarithm of the variable.)

To use these results, $D_{LT}$ and $V_{D_{LT}}=\sqrt{\text{Var}[D_{LT}]/D_{LT}}$ are evaluated with Eqns. 5–6. Corresponding statistics of $\Delta$ are assumed here to be

$$E[\Delta] = e^{2\sigma_{\log N}} = 10^{2\sigma_{\log N}}; \quad V_{\Delta} = 0.7$$

This result for $E[\Delta]$ is straightforward: the design $S$–$N$ curve is shifted conservatively by 2 standard deviations ($2\sigma_{\log N}$) on log scale. (For the Class E $S$–$N$ curve used here, $\sigma_{\log N}=0.2509$ so that $E[\Delta]=3.18 [10]$.)

The assumption that $V_{\Delta}=0.7$ is more tenuous. This variability must include not only the scatter in the $S$–$N$ curve, but all other uncertainties (e.g., Miner’s rule that generalizes from constant to variable amplitude loads). The value $V_{\Delta}=0.7$ is roughly consistent with values suggested in the literature (e.g., [11, 12]), and hence is adopted here. For highly uncertain fatigue loading processes, this value may need to be increased2.

Numerical Results

We now apply these results, using long-term probabilities $p_i$ of different current profiles from a deepwater Gulf of Mexico site. The long-term damage rates, with and without software

1The mean and standard deviation values are only plotted at the (logarithmic) midpoint of each bin—and a straight line connected for visual purposes between these points. The bin widths are constant on log scale.

2The converse benefit, if $V_{\Delta}$ is increased beyond 0.7, is that the effect of software uncertainty (reflected through $V_{D_{LT}}$) becomes relatively less important.

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correction, are found to be

\[
\dot{D}_{LT, \text{pred}} = \sum_i d_i = 3.9 \times 10^{-2} = \frac{1}{26 \text{ yrs}} \tag{10}
\]

\[
\dot{D}_{LT} = \sum_i d_i \rho_i = 1.0 \times 10^{-5} = \frac{1}{1.0 \times 10^5 \text{ yrs}} \tag{11}
\]

There is thus a significant effect of software bias—the corrected damage rate is reduced by more than 3 orders of magnitude:

\[
\text{Mean software bias } = \bar{B} = \frac{\dot{D}_{LT}}{\dot{D}_{LT, \text{pred}}} = \frac{1.0 \times 10^{-5}}{3.9 \times 10^{-2}} = 2.5 \times 10^{-4} \tag{12}
\]

To understand this, note that most of the significantly damaging currents have \(\dot{D}_{\text{pred}}\) values in the 0.1–3.0 \([\text{yr}^{-1}]\) range. In this range, Fig. 2 indeed shows mean biases on the order of \(10^{-3}–10^{-4}\).

Table 1 shows corresponding estimates of \(V_{\Delta}\), the coefficient of variation of \(\dot{D}_{LT}\) due to software errors. Results are shown for 3 different (hypothetical) values of \(\rho\), the correlation coefficient between errors in different \(D_{\text{pred}}\) bins. Notably, these values (between .05–.14) are considerably smaller than the “background” variability of \(V_{\Delta}=0.7\). In view of the SRSS operation in Eqn. 8, the effect of \(V_{\Delta}\) will therefore be rather negligible. This is shown in Table 1: \(V_{T_f}\) varies only from .702–.713.

**The Choice of Probabilistic Model**

Figure 3 fits a Weibull model of \(T_f\), based on its moments in Table 1. As shown in the table, the COV values are based on the various assumed values of the correlation coefficient \(\rho\). The Weibull model is often used to model failure times of deteriorating systems, as its hazard function increases monotonically (as a power-law function) for \(V_{T_f} < 1\). It also arises as a minimum strength distribution, hence is relatively heavy in its lower (left) tail. As this is the tail of interest (low failure times due to rare weak members), this would seem a fairly reasonable choice.

Figure 4 compares Weibull, Gamma, and lognormal models fit to the same moments. (The correlation \(\rho=1\) case is assumed.) As may be expected, the Gamma model predicts somewhat lower

<table>
<thead>
<tr>
<th>Case:</th>
<th>(E[\Delta])</th>
<th>(V_{\Delta})</th>
<th>(\dot{D}_{LT}) ([\text{yr}^{-1}])</th>
<th>(E[T_f]) ([\text{yrs}])</th>
<th>(\rho)</th>
<th>(V_{D_{LT}})</th>
<th>(V_{T_f})</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIV, Gulf of Mexico MODU drilling riser</td>
<td>3.18</td>
<td>0.7</td>
<td>(1.02 \times 10^{-5})</td>
<td>310000</td>
<td>0.0</td>
<td>0.051</td>
<td>0.702</td>
</tr>
<tr>
<td>SHEAR7: Ver 4.6, default parameters</td>
<td>0.5</td>
<td>1.0</td>
<td>(0.137)</td>
<td>0.708</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S–N: DNV Curve E</td>
<td>3.74</td>
<td>(3.74 \times 10^{-2})</td>
<td>85</td>
<td>0.0</td>
<td>0.108</td>
<td>0.708</td>
<td></td>
</tr>
<tr>
<td>VIV, North Sea MODU drilling riser</td>
<td>3.18</td>
<td>0.7</td>
<td>(3.74 \times 10^{-2})</td>
<td>85</td>
<td>0.0</td>
<td>0.108</td>
<td>0.708</td>
</tr>
<tr>
<td>SHEAR7: Ver 4.6, default parameters</td>
<td>0.5</td>
<td>1.0</td>
<td>(0.165)</td>
<td>0.719</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S–N: DNV Curve E</td>
<td>3.74</td>
<td>(3.74 \times 10^{-2})</td>
<td>85</td>
<td>0.0</td>
<td>0.108</td>
<td>0.708</td>
<td></td>
</tr>
<tr>
<td>WIF plus VIV, Jackup production riser</td>
<td>2.11</td>
<td>0.7</td>
<td>(1.94 \times 10^{-3})</td>
<td>1100</td>
<td>0.0</td>
<td>0.106</td>
<td>0.708</td>
</tr>
<tr>
<td>Flexcom plus SHEAR7(v4.6 default)</td>
<td>0.5</td>
<td>1.0</td>
<td>(0.150)</td>
<td>0.716</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S–N: DNV High-Strength with C.P.</td>
<td>3.74</td>
<td>(3.74 \times 10^{-2})</td>
<td>85</td>
<td>0.0</td>
<td>0.108</td>
<td>0.708</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 1.** Results across various cases.
failure probabilities than the Weibull (with its heavy left tail). More dramatic is the lognormal result: it predicts a failure probability of $10^{-16}$ only when service lives approach 1000 years—at which time the Weibull and Gamma models yield failure probabilities 13 and 12 orders of magnitude larger, respectively.

This suggests that it may be imprudent to use a lognormal model in these applications, where capacity uncertainty dominates and the lower tail (weakest strength, smallest failure time) is of critical interest. The lognormal, due to its exponential transformation, assigns relatively large probabilities in its upper (right) tail, and hence is quite narrow in its lower tail. This point is fairly clear in the PDF comparison in Fig. 5. In view of the popularity of the lognormal model in fatigue applications, this point should perhaps be emphasized.

**VIV of a North Sea Drilling Riser**

We consider now a riser subjected to VIV in a North Sea environment. Data have been measured for a drilling riser, without strakes or any other VIV suppression device. Measured damage rates are inferred from measured riser accelerations. Predicted damage rates are found from version 4.6 of SHEAR7, again us-
ing default parameters and the DNV Class E S–N curve.

Figure 6 compares measured and predicted damage rates (per year) for 234 10-minute data segments. Results are rather better behaved than in Fig. 1: the data show stronger correlation, and no distinct subpopulations. Nonetheless, there is considerable scatter, and predictions show a clear conservative bias.

Figure 7 shows the result of binning data based on the predicted damage level $D_{\text{pred}}$. Results are qualitatively similar to those in Fig. 2: the bin average and standard deviation are numerically similar, and both fall off (showing increasing conservatism in SHEAR7) as $D_{\text{pred}}$ grows.

We again apply the reliability analysis steps in Eqns. 5–9, now with long-run $p_i$ appropriate for a set of North Sea current profiles. The long-term damage rates, again with and without software correction, are

$$D_{LT,\text{pred}} = \sum_i d_i = 4.9 \times 10^{-1} = \frac{1}{2.0 \text{ yrs}}$$

$$\dot{D}_{LT} = \sum_i d_i m_i = 3.7 \times 10^{-2} = \frac{1}{27 \text{ yrs}}$$

Note that the software conservatism here is only an order of magnitude, compared with 3 or more orders of magnitude in the Gulf of Mexico case:

$$\text{Mean software bias} = B = \frac{\dot{D}_{LT}}{D_{LT,\text{pred}}} = \frac{3.7 \times 10^{-2}}{4.9 \times 10^{-1}} = 0.076$$

Note that there are markedly different current environments in the two cases. The Gulf of Mexico example is in very deep water marked by steeply sheared currents, while the North Sea case is in shallower water with slab-like currents. These may therefore impose different predictive challenges to a code like SHEAR7, and hence lead to different biases. Finally, Fig. 8 shows the resulting variation of failure probability with service life in this North Sea case.

APPLICATION TO WAVE-INDUCED FATIGUE

Finally, we consider the fatigue of production risers on a jackup, at a water depth of roughly 100m. The motions of a number of risers have been measured. These have been used to compare fatigue damage predictions with the damage inferred from the measurements across different conditions.

Some features of these data:

1. Predicted fatigue damage is the sum of wave fatigue damage calculated in Flexcom and VIV fatigue damage calculated in SHEAR7v4.6. The wave-induced damage typically dominates in this case.

2. “Measured” fatigue damage is inferred from accelerometer measurements on several risers, based on a transfer function approach developed in BPs Upstream Engineering Centre (UEC).

3. The location of interest is a particular fatigue detail on a component.

4. In both the predicted and measured cases, damage is conservatively assumed to accumulate at a single location on the component circumference.

5. Fatigue damages are calculated using the DNV HS (high-strength) S–N curve with Cathodic Protection. From Section 2.4.10 of [13], the design S–N curve is log $N = 17.446 - 4.70 \log S$, while the mean S–N curve is log $N = 17.770 - 4.70 \log S$. It follows that $E[\Delta] = 10^{(17.770 - 17.446)}$ or 2.11.

6. Predictions have been made based on a fixed upper boundary condition, which corresponds better with measured data when compared to a pinned condition.

Numerical Results

Figure 9 compares predicted and measured fatigue damage (per 30 minutes), and Fig. 10 shows the resulting modelling error $X$. Trends are similar to other cases: increasing conservatism (lower $X$ values) as the predicted damage $D_{\text{pred}}$ increases. As may be expected, the conservatism (in this wave-induced fatigue case) is not as large as in the VIV Gulf of Mexico case.

The long-term damage rate, $D_{LT}$, is again estimated from Eqn. 5. The only conceptual difference is that for wave-induced fatigue, the $p_i$ denote probabilities that various $(H_S, T_P)$ pairs occur (rather than current profiles, as in the VIV case). For our long-term predictions, we consider wave-induced fatigue only.
FIGURE 9. Measured damage rate versus predicted fatigue damage rate (per 30-minute interval) for a jackup riser. Predictions include both wave-induced fatigue and VIV.

FIGURE 10. Modelling error versus predicted fatigue damage rate (per 30-minute interval) for a jackup riser. Predictions include both wave-induced fatigue and VIV.

The long-term damage rates, again with and without software correction, are

$$D_{LT,\text{pred}} = \sum d_i = 1.8 \times 10^{-7} \text{ per 30-min} = \frac{1}{320 \text{ yrs}}$$  \hspace{1cm} (16)

$$\dot{D}_{LT} = \sum d_i \Delta \tau = 1.1 \times 10^{-7} \text{ per 30-min} = \frac{1}{510 \text{ yrs}}$$ \hspace{1cm} (17)

As noted above, the the software conservatism here is consider-

ably less than in the VIV cases:

$$\text{Mean software bias } = B = \frac{\dot{D}_{LT}}{\dot{D}_{LT,\text{pred}}} = \frac{1.1}{1.8} = 0.61$$ \hspace{1cm} (18)

Finally, Fig. 11 shows failure probability estimates for various values of service life $T$. Results are shown both for total failure probability over the life, and worst annual failure probability (in the last year of service).

FIGURE 11. Failure probability versus service life $T$ for the jackup riser. Weibull and Gamma models of $T_f$ assumed, with $\rho$=1.

ESTIMATION OF LOAD FACTORS

The foregoing results are all problem-specific—the absolute numbers are tied directly to the situation at hand. In this final sec-

tion, we seek to produce more generalized results. In particular, from a design perspective, we seek to give insight into plausible values of the safety factor $\gamma = T_{des}/T_{op}$, the ratio between the de-

sign life $T_{des}$ and the operating life $T_{op}$ of the fatigue-sensitive component. To anchor the ensuing discussion, $\gamma=10$ is a typical value used in this context.

In probabilistic design, we will typically seek to impose a target value on the failure probability

$$p_F(T_{op}) = P[\dot{D}_{LT}T_{op} > \Delta]$$ \hspace{1cm} (19)

It is clear from the foregoing results that it is critical to estimate statistics of the software bias $B$:

$$B = \frac{\dot{D}_{LT}}{\dot{D}_{LT,\text{pred}}}$$ \hspace{1cm} (20)
To incorporate Eqn. 20 into Eqn. 19, we note that since \( T_{des}=1/D_{LT, pred} \):

\[ D_{LT}T_{op} = BD_{LT, pred}T_{op} = B \frac{T_{op}}{T_{des}} = \frac{B}{\gamma} \quad (21) \]

Substituting Eqn. 21 into Eqn. 19,

\[ p_F(T_{op}) = P \left[ \frac{B}{\gamma} > \Delta \right] = P \left[ \frac{\Delta}{B} < \frac{1}{\gamma} \right] \quad (22) \]

The latter form of this result is most convenient for our purposes, as it collects all uncertain terms into a “capacity” variable \( \Delta/B \) that should not exceed \( 1/\gamma \). A normalized version of this variable is defined as

\[ U = \frac{\Delta/B}{\gamma} \quad (23) \]

in which \( \bar{A} \) and \( \bar{B} \) are the mean values of the respective variables. \( U \) will thus have (approximately) unit mean. Recasting Eqn. 22 in terms of \( U \) yields

\[ p_F(T_{op}) = P \left[ U < \frac{\bar{B}}{\gamma \bar{A}} \right] \quad (24) \]

The quantity \( \bar{B}/\gamma \bar{A} \) serves as an (inverse) reliability index—smaller values yield higher reliability. Thus, higher reliability is achieved by (1) reducing the (unconservative) software bias \( \bar{B} \), (2) increasing the (conservative) bias \( \bar{A} \) in the \( S-N \) curve, or (3) increasing the safety factor \( \gamma \).

Figure 12 shows typical results from Eqn. 24. A Weibull distribution has been assumed for \( U \), with \( \gamma=0.8 \) (\( V \) here is the same as \( V_{T_f} \) in the previous results.) \( \bar{A}=3 \) has been assumed, an intermediate value between those for the F2 and E \( S-N \) curves. Bias values \( \bar{B}=1.0, 0.1, 0.01 \), and 0.001 have been assumed.

It is clear that the software bias \( \bar{B} \) plays a critical role here. For our Gulf of Mexico case we find \( \bar{B} \) values less than .001 (Eqn. 12). Using a Weibull model for \( T_f \) in this case, even with no safety factor (\( \gamma=1 \)) we have \( p_f(T_{op}) \) on the order of \( 10^{-6} \), well within the highest DNV safety class. (DNV permits \( p_f=10^{-5} \) as the worst annual failure probability, which in turn permits a larger \( p_f \) over the entire operating life.) In contrast, if the bias increases to \( \bar{B}=0.1 \), one cannot achieve \( p_f(T_{op})=10^{-5} \) even if \( \gamma=100 \).

Another way to consolidate our foregoing results is to consider a plot of the Weibull CDF (cumulative distribution function) of \( T_f \) versus the normalized variable \( T_{op}/E[T_f] \) (analogous to the PDF plot in Fig. 5). The explicit form of this CDF is shown in Appendix I (Eqn. 27). Figure 13 shows the resulting CDF for COV values of 0.7 and 0.8, which bracket the cases shown here. Note that this same normalized curve applies for all our cases, when entered at the appropriate value of \( T_{op}/E[T_f] \). The figure shows these values for our 3 cases, with \( T_{op}/E[T_f] \) levels for the three cases considered here.

FIGURE 12. Safety factor \( \gamma \) versus failure probability \( p_f \) over operating life. Weibull distribution assumed with \( V=0.8 \).

FIGURE 13. Cumulative distribution function of Weibull failure time \( T_f \) versus normalized time \( T_{op}/E[T_f] \). Also shown are \( T_{op}/E[T_f] \) levels for the three cases considered here.
Finally, if we wish to use Fig. 13 in a design context to evaluate $p_f(T_{op})$, it is useful to note that

$$\frac{T_{op}}{E[T_f]} = \frac{B}{\gamma \Delta} \quad (25)$$

This result is derived briefly in Appendix II. It implies that the horizontal axis of Fig. 13 can be equivalently viewed as $B/\gamma \Delta$.

Consider, for example, a typical wave-induced fatigue (WIF) case designed with $\gamma=10$, $B=1$ (unbiased software), and $\Delta=3$ (between E and F2 $S-N$ curves). In this case, the critical level $B/\gamma \Delta$ is 1/30 or about 0.03. This is close to the values found for both our WIF and VIV NS cases (Fig. 13). Substantially higher reliability can be achieved only if the software has significant conservative bias (as in the VIV GOM case), or if the safety factor $\gamma$ is increased.

SUMMARY AND CONCLUSIONS

We have presented a method to estimate fatigue reliability, using measured data to calibrate statistical uncertainty in fatigue software predictions. Applications to both VIV and wave-induced fatigue have been shown.

The method shown here is a considerable extension of our previous work (e.g., [9]). First, a non-parametric model has been introduced, to reflect different software errors $X$ for different levels of predicted damage $D_{pred}$. This is found useful to capture the general trend of increasing conservatism (lower $X$ values) for higher $D_{pred}$ (e.g., Fig. 2, Fig. 7, and Fig. 10). As a second modelling extension, we have included a $\sqrt{n_{ij}}$ term in the $\Delta$ equation. This reflects our interest here in the variability of the long-run damage rate, not of individual damage rates in 10–30-minute segments. Finally, we have compared the effect of different probability models—e.g., Figs. 4–5—and concluded that the lognormal is likely over-optimistic in these capacity-driven fatigue applications.

The software bias $B$ plays a critical role here. The software conservatism is greater for VIV applications than wave-induced fatigue—and this conservatism is especially large in the Gulf of Mexico example (see Eqn. 12 vs Eqn. 15 vs Eqn. 18). Using a Weibull model for $T_f$ in this case, even with no safety factor ($\gamma=1$) we have $p_f(T_{op})$ on the order of $10^{-6}$, well within the highest DNV safety class. In contrast, if the bias increases to $\frac{B}{\gamma}=0.1$, one cannot achieve $p_f(T_{op})=10^{-5}$ even if $\gamma=100$ (Fig. 12).

Of course, it should be noted that these methods depend directly on the quality of the data collected. Therefore, great care should be taken to assure the accuracy of the hardware and the calculation methodology used to infer fatigue damage.

REFERENCES

APPENDIX I: WEIBULL PROBABILITY MODEL

Since it forms the basis of much of our work here, we briefly describe some properties of the Weibull model. It is convenient to relate a Weibull random variable, \( W \), to a standard (unit-mean) exponential random variable \( E \):

\[
W = cE^p
\]  

(26)

Here \( c \) and \( p \) are distribution parameters. Since \( P[E < x] = 1 - \exp(-x) \), the cumulative distribution function of \( W \) is

\[
F_W(w) = P[W < w] = 1 - \exp[-(w/c)^{1/p}]
\]  

(27)

This result, with suitable choice of parameters \( c \) and \( p \), has generated the failure probability curves in Fig. 3, Fig. 8, and Fig. 11.

It remains to relate the parameters \( c \) and \( p \) to moments of \( W \). For the standard exponential variable \( E \), the expected value of \( E^p \) is simply \( p! \), or \( \Gamma(p+1) \) for non-integral \( p \). Therefore,

\[
E[W] = cp! \rightarrow c = \frac{E[W]}{p!} = \frac{E[W]}{\Gamma(p+1)}
\]  

(28)

The unitless parameter \( p \) is in turn related to \( V_W \), the coefficient of variation of \( W \):

\[
V_W = \left[ \frac{E[W^2]}{E[W]^2} - 1 \right]^{1/2} = \left[ \frac{(2p)!}{p^{1/2}} - 1 \right]^{1/2}
\]  

(29)

This is an implicit result for \( p \), which in general requires numerical solution. Fortunately, a simple approximation is available:

\[
V_W \approx p; \quad 0 \leq p \leq 1.5
\]  

(30)

Figure 14 shows that agreement is excellent for \( 0 \leq p \leq 1.5 \), which covers most practical cases of interest. Thus, in practice one first uses Eqn. 30 to estimate \( p \), and then Eqn. 28 to find \( c \).

APPENDIX II: DERIVATION OF EQUATION 25

First note that we can use Eqn. 21 to solve for \( \dot{D}_{LT} \):

\[
\dot{D}_{LT} = \frac{B}{\gamma T_{op}}
\]  

(31)

Substituting this result into Eqn. 7 yields

\[
T_f = \frac{\Delta}{D_{LT}} = \frac{\Delta T_{op}}{B}
\]  

(32)

Using the same approximations applied previously, the mean value of \( T_f \) can be estimated as

\[
E[T_f] = \frac{\Delta T_{op}}{B}
\]  

(33)

Algebraic manipulation of Eqn. 33 yields Eqn. 25.