Application of Measured Loads to Wind Turbine Fatigue and Reliability Analysis

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Application of Measured Loads to Wind Turbine Fatigue and Reliability Analysis

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Background

Fatigue loadings on wind turbines are fairly difficult to characterize because they are of variable amplitude with the intensity of the variations depending on the wind environment of the turbine. The loadings must be comprehensively described to conduct fatigue analyses for various components. Therefore, the loads at many locations on a turbine must be determined (either from analysis or test) and archived for future use in the fatigue analysis. There is a need for a procedure that describes loads simply, while relying on a fairly small set of parameters described over all wind conditions (wind speed and turbulence). This procedure should be capable of including information on how well the loads have been determined, i.e., the uncertainty in the knowledge of the loads.

There are a few universally applied procedures currently in practice for describing fatigue loads on wind turbines. First, the loading time series is obtained either from prototype measurements or computer simulation. The time series is then rainfall counted to identify significant cycles that produce fatigue damage. Rainflow counting is a procedure for determining the damaging loading cycles (mean and amplitude) in an irregular time series (SAE, 1988). Cycles are usually summed into bins referenced to the mean and amplitude of the cycle. The end result is a histogram of the number of occurrences of cycles in each load mean and amplitude bin, which condenses the data in the original time series—by factors of thousands. The cost of the condensation is that the resolution of the data is reduced to the bin size of the histograms. IEA fatigue recommendations suggest a minimum of 50 bins (IEA, 1990). However, any particular sample might use only a fraction of the 50 available bins and resolution can be reduced beyond what is necessary for future analysis. Finally, the distributions are described as a function of average wind conditions determined over a short interval, typically ten minutes.

Figure 1 shows a typical description of the cycle amplitudes and means in a particular wind speed interval. The plot was produced using the rainflow analysis features of the LIFE2 code (Schluter and Sutherland, 1989). LIFE2 does fatigue analysis based on these histogram-type descriptions of loadings, one loading description for each wind speed interval covering the entire operating range. Separate distributions cover start-stop transients and buffeting while parked in high winds. Because the variation in the mean of each range is often a minor factor in the damaging potential of the loadings, this paper assumes the mean can be treated as a constant, and focuses on the distribution of amplitudes only. Figure 2 shows the same data plotted as a function of amplitude only. If the mean values are close to the ultimate strength, this simplification will lead to significant errors, but that situation should be fairly rare for well designed components.

The measured histogram is often taken to be the characteristic distribution of loading cycles at the wind conditions of the measurement (or simulation). This is an approach that could be called “data based” or “non-parametric” in that the definition of the distribution of load amplitudes is based on the measured (or synthesized) data. Then the histograms are commonly used directly to calculate the fatigue lifetimes of components. Sutherland, for example, applies this approach directly in the LIFE2 fatigue and fracture analysis code (Sutherland, 1989). This approach has the advantage of simplicity; there is no need for distribution modeling. However, it depends on a rather large set of data to describe each loading environment and does not lend itself readily to illustrating systematic trends across wind conditions. We also seek to understand the importance of loads beyond the measured range and include them in the analysis when found important.

An alternate approach which could be called “statistical” or “parametric” is to calculate a few statistics of the loading and use those statistics to describe the loading distribution. Strictly speaking, a histogram is a collection of statistics, where the
relative frequency of each histogram cell is a parameter, or statistic, of the distribution. This leads to a set of about 50 separate statistics to describe the complete discretized distribution. Statistical approaches usually seek to condense the description by calculating a very small set of descriptive statistics. They have the drawback of only being as good as the assumed parametric form and can be overly restrictive as a result. On the good side, by condensing the number of descriptive parameters, they promote understanding, illustrate systematic variations and trends, and permit smooth extrapolation where data are missing. As always, care should be taken when extrapolating measured statistics beyond the range of the measurement program, especially if measurements are taken in one regime and the extrapolation is into another regime (such as from linear aerodynamics into stalled operation). One should never extrapolate too far from the measured data.

Highly condensed statistical approaches are not new. Veers (1982) proposed the use of Rayleigh distributions of stress amplitudes, which rely on only the RMS of the stress histories to describe the entire distribution. The problem is that the Rayleigh distribution appears to be appropriate only for a single location on a single type of wind turbine (flatwise loads on vertical axis wind turbines). Jackson (1992) proposed a scheme based on an exponential fit to loading amplitudes from relatively short data sets from horizontal axis wind turbines. Kelley (1993) continues in this vein emphasizing the exponential nature of the low cycle, high stress (LCHS) tail of the distribution. In this approach, only the slope of an exponential fit to the highest of the cycle amplitudes is used to describe the entire distribution. It is not yet clear just where the fit should start, nor that the exponential distribution is always the appropriate choice.

Recently, Ronold, et al. (1994) and Lange and Winterstein (1996) used a method for organizing loads based on the moments of the measured load amplitudes. Successive moments are particularly descriptive of the load distribution: the first moment is the mean, the second moment describes the spread about the mean, and the higher moments reflect more detail in the tail behavior of the distribution. The moments are then functionally fitted to both wind speed and turbulence intensity. The actual distribution of stress amplitudes at any given wind condition can be estimated from the moments as described in Winterstein and Lange (1995). When dealing with bimodal distributions, ones with two peaks in the frequency of occurrence, the loads must be processed to eliminate the lower-load peak or the three moment model will not be able to match the data. Retaining only the higher peak has been shown to account for all the damaging potential of the loading (Winterstein and Lange, 1995).

The main purpose of the Ronold et al. and Lange and Winterstein papers, however, was to show how to evaluate safety factors needed to produce a predetermined level of risk of fatigue failure. Explanations were aimed at illustrating the uncertainty in the stress distributions due to limited data. So, the details of calculating the statistical quantities and using them to describe the load distributions over all climate conditions was given secondary importance in the presentations. Therefore, the advantages of this approach may not be clearly evident from the existing literature. The purpose of this paper is to illustrate the methods developed previously and to show why this statistical approach is likely to accomplish the needs of fatigue-life prediction, loading-spectra definition, and uncertainty analysis.

Using Moments of Load Amplitudes to Describe Fatigue Loading

The statistical moments of random quantities are characteristic values that can be used to approximate their distribution functions. The first three moments, $\mu_i$, of the rainflow-range amplitudes, $S$, are defined here as:

$$\mu_1 = E[S]$$

$$\mu_2 = \frac{\sigma^2}{\mu_1} \quad \sigma^2 = E[(S - E[S])^2]$$

$$\mu_3 = \frac{E[(S - E[S])^3]}{\sigma^3}$$

where $E[\cdot]$ is the expectation (or average) operator. The first moment is the mean or average amplitude, a measure of central tendency. The second moment is the Coefficient of Variation (COV), which is the standard deviation divided by the mean, a measure of the distribution spread. The first two moments can be exactly matched by any two parameter distribution, and are often fitted with the Weibull (of which the exponential and Rayleigh are special cases with COV of 1.0 and 0.523, respectively). The third moment is the skewness, which provides more detailed information on the tail behavior of the distribution. Since load amplitude data are often well fit by a Weibull distribution, a slight distortion of the Weibull distribution is used to exactly match the first three statistical moments (Lange, 1996). The three-moment match produces a distortion of the standard Weibull distribution function so that it plots as a quadratic rather than linearly on a Weibull plot.

An example data set will be used here to illustrate the procedure for analyzing fatigue-loading data to produce a comprehensive load definition over all wind conditions. The data displayed here were collected from the Advanced Wind Turbines’ AWT-26 P2 prototype in Tehachapi, California in 1994. They are perhaps a typical example of data collected on prototype turbines during development efforts around the world. These data are from a single location on the turbine—the blade root flatwise bending moment.

![Fig. 2 Typical load amplitude histogram for a HAWT flatwise bending moment](https://example.com/fig2.png)
wise bending—but could be from any component of loading with fatigue damaging potential. The AWT-26 turbine is rated at 300 kilowatts and is a 2-bladed, downwind, teetered, free yaw, fixed pitch, stall controlled, constant speed, horizontal-axis wind turbine. The rotor is 26 meters (m) in diameter. The normal operating speed is 57 revolutions per minute (rpm’s).

The data consist of over thirty hours of turbine operation collected in ten-minute segments.

Figure 3 shows the number of ten minute samples that fall into each wind bin divided over both wind speed and turbulence intensity, defined as standard deviation of wind speed divided by mean wind speed. Wind speed runs from about 5 to 20 m/s and turbulence intensity ranges from about 8 to 30%, although most of the samples fall on the lower half of that range. Figure 3 illustrates one of the difficulties of determining the long-term loading spectrum directly from measured data even with a large sample. The measurements are rarely indicative of the test-site distribution of climate conditions, much less of any particular site for which the turbine is likely to be installed. Like most measurement campaigns, the data are sampled more heavily in high wind conditions where the turbine response is more interesting and provides information on high wind response. Simply including all the measurements into a global distribution would not produce a loading spectrum indicative of any site. The data should be used to determine how the turbine responds as a function of wind conditions and then it can be applied to any site for which it might be intended, including standard type classification sites in certification standards.

Within each wind bin, histograms of rainfall amplitudes have been combined from all the ten-minute samples that share the bin characteristics for average wind speed and turbulence intensity. Figure 4 shows the load amplitude data at one particular wind condition, 11.5 m/s wind speed and 0.155 turbulence intensity, on a Weibull probability scale. This scale enhances the tail of the distribution where much of the fatigue damage is caused. A quadratic Weibull fit created to match the first three moments of the load amplitude data is superimposed on the plot. Distribution shapes can be similarly approximated at any wind speed bin from the moments of the amplitudes in that bin.

Fitting Moments of the Rainflow Amplitudes to Wind Conditions. The moments of the rainflow-range amplitudes were calculated for all the 30-plus hours of data. Figures 5, 6, and 7 show the results for the mean, COV, and skewness, respectively. There appears to be an upward, approximately linear trend of the mean with wind speed, a mild tendency for COV to decrease with wind speed, and no particular trend of skewness with wind speed.

Moment behavior as a function of wind conditions is illustrated by a standard regression fit of the moment data over the two dimensional space of wind speed, \( V \), and turbulence intensity, \( I \), with the following functional form.

\[
E[\mu_i] = a_i \left( \frac{V}{V_{\text{ref}}} \right)^{b_i} \left( \frac{I}{I_{\text{ref}}} \right)^{c_i}
\]

in terms of the individual mean wind speeds, \( V_i \), observed in each 10-minute segment. In this example \( V_{\text{ref}} = 11.4 \text{ m/s} \) and the analogous geometric mean of the turbulence intensity is \( I_{\text{ref}} = 0.157 \). We denote these estimates of the parameters, \( a_i, b_i \), and \( c_i \), by \( \hat{a}_i, \hat{b}_i \), and \( \hat{c}_i \) to distinguish them from the true (but unknown) values. Note also that by normalizing our fits by the geometric means \( V_{\text{ref}} \) and \( V_{\text{ref}} \) of the data, our estimate of the
leading term (strictly, $\ln (\hat{a}_i)$) is uncorrelated with our estimated slopes $\hat{b}_i$ and $\hat{c}_i$ (Neter et al., 1996; Eqs. 6.45-6.48).

In addition to the estimates $\hat{a}_i$, $\hat{b}_i$, and $\hat{c}_i$, a standard regression analysis provides several other useful pieces of information. These include the corresponding standard deviations of the estimates, $\sigma_{\hat{a}_i}$, $\sigma_{\hat{b}_i}$, and $\sigma_{\hat{c}_i}$, which reflect the effect of limited data. These are commonly reported in normalized form by associated "$t$-statistics," which are the inverse of the COV definition:

$$t_{\hat{a}_i} = \frac{\hat{a}_i}{\sigma_{\hat{a}_i}}$$

and similarly for $t_{\hat{b}_i}$ and $t_{\hat{c}_i}$. Large $t$ values indicate relatively important parameters; i.e., parameter estimates that are "significantly" different from zero, as compared with their statistical uncertainty. One may, for example, regard variables with $|t| > 2$ as statistically significant, since if the true $a_i$ = 0, the observation $t_{\hat{a}_i}$ = 2 corresponds to the improbable event that the estimate $\hat{a}_i$ happens to fall 2 standard deviations away from its mean.

Finally, regression also supplies a gross measure of the adequacy of the fit in Eq. 4. This is commonly reported as the dimensionless quantity $R^2$, the fraction of the variability "explained" by the predictive equation. In this case, because linear regression is applied to the logarithm of Eq. 4, $R^2$ is computed as

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (\ln \mu_i - \ln \bar{\mu}_i)^2}{\sum_{i=1}^{N} (\ln \mu_i)^2}$$

Here $\mu_i$ is the observed moment value computed directly from the data, while $\bar{\mu}_i$ is the corresponding estimate obtained from Eq. 4 with its estimated parameters $\hat{a}_i$, $\hat{b}_i$, and $\hat{c}_i$. $R^2 = 1$ implies perfect prediction; i.e., $\mu_i = \bar{\mu}_i$ for all observations. Table 1 summarizes all the mean parameter values, $t$ values, and $R^2$ values of each parameter for each moment.

<table>
<thead>
<tr>
<th>Table 1 Load-amplitude moment vs. wind condition-regression results</th>
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Example. Figure 8 shows the functional fits to all three moments versus wind speed at the characteristic turbulence intensity, $I_{ct}$.

The $t$ values of the parameters reflect how confident you can be in a nonzero value of the coefficient. Since the $a_i$ reflect the regression fit at the reference conditions, there should be high confidence in their values. For the exponents, a zero value means no dependence of the moment on the particular variable, either $V$ or $I$. All of the coefficients in this example exhibit high levels of significance, except for a clear lack of dependence of skewness on $I$. Most of the variability in the mean and COV is explained by the regression, as indicated by relatively high $R^2$ values of 0.77 and 0.86. But the regression explains very little of the skewness variations ($R^2 = 0.26$), which indicates that there is a lot of sample to sample variation that is uncorrelated with either $V$ or $I$.

Reasons Why the Definition of $I$ Used Here May Not Be Best. It would seem from Table 1 that $I$ as calculated here is not necessarily the best defining factor in segregating stress responses at the same average wind speed. From a physical point of view, one would expect that some measure of the roughness in the inflow must affect the stress amplitude distributions. There are many reasons why $I$ may not be adequate to describe it. The greatest deficiency is probably that it does not reflect any of the spatial variations in the flow. Several researchers have reached that conclusion and are proposing better measures of inflow damaging potential. Kelly (1993) has suggested measures of atmospheric stability and shear stress, which should have substantial influence on the spatial distribution of wind speed fluctuations. Barnard and Wendell (1997) suggest using
two point measurements to directly measure the spatial variations in the wind. Both require additional measurements of either temperature, all three wind components, or wind speed at additional locations, which are not always available. However, the additional measurements may ultimately be required. Here, \( I \) was estimated from the standard deviation of the wind speed over each ten-minute interval with no additional processing. Connell et al. (1988) have noted that calculations of \( I \) should be done with some sort of “de-trending,” or high-pass filtering that will remove the long term fluctuations while preserving the variations likely to drive rotor dynamics. Indeed, high pass filtering of \( I \) has been shown to improve the ability of the resulting regression to explain the moments of the rainfall range distributions in at least some wind turbines (Kashef and Winterstein, 1998). \( R^2 \) values increased markedly, well beyond what was possible with simple detrending. It may be that some sort of processed turbulence measure such as this will be required to make the atmospheric statistics truly descriptive of the machine dynamic response. However, that is a topic well beyond the scope of this work.

**Loading Cycle Rate:** The rate at which cycles are accumulated is also an important quantity in conducting a fatigue analysis. The cycle rate can be treated just like the moments of the load amplitudes in the previous section. Figure 9 shows the AWT cycle rate data plotted versus wind speed. Again, for this example, there is minimal dependence on \( I \), and significant dependence on \( V \). However, the relative size of the change in cycle rate with wind speed is small enough (\( \pm 15\% \)) that variations in the rate will have a minimal effect on lifetime estimates.

### Using the Loading Model in Fatigue and Reliability Analysis

Because the trends with turbulence intensity are small in this data set, we will restrict the rest of the loading descriptions in this example to wind speed dependence only. Analysis including the two dimensional regression has been published by Lange and Winterstein (1996) and by Ronold et al. (1994). The plotting is simplified and perhaps the approach may be more clearly demonstrated by restricting the example to one dimension, \( V \).

Once the moments have been described over all wind speeds by Eq. 4, the loading distributions can be estimated using the procedures described in detail in the above references. Figure 10 shows the resulting load amplitude distributions, plotted as exceedence diagrams, for several wind speeds. These wind speeds reflect the short-term (10 minute) average typically used in data gathering. The shapes are quite similar especially due to the fact that the COV and skewness (second and third moments) depend only weakly on wind speed (see Fig. 8).

With the load distributions defined conditionally on the wind speed, it is a fairly simple matter to determine the long-term load distribution, which is sometimes called the design spectrum. It is calculated by integrating the conditional distributions over all wind speeds.

\[
F(S) = \int_0^\infty F(S|V)f(V)dV
\]

where \( f(V) \) is the wind speed probability density function and \( F(S|V) \) is the distribution of load conditional on wind speed. \( F(\cdot) \) could be either the density function, the cumulative distribution function, or the inverse cumulative distribution function (which is the same as the exceedence diagrams shown in Fig. 10). Cut-in and cut-out conditions can also be applied by integrating between the limits. Figure 11 shows design spectra in terms of exceedence diagrams calculated from Eq. 6 for Rayleigh distributed wind speeds with two different long-term averages, 6 and 7 m/s. The two spectra are quite different in shape from any of the short-term distributions in Fig. 10. The effect of different sites is readily seen as about a factor of three difference in the probability (frequency of occurrence) for a given load amplitude in the high amplitude end of the plot in Fig. 11. The fatigue damage is then calculated directly from the long-term distribution and the loading frequency.

The advantages of describing the loading first conditionally on wind conditions and then applying the conditional definition to site-specific wind distributions is firstly that the significance of climatic conditions can be determined. Parametric studies are easily accomplished by varying the wind speed distributions (or, if included in the analysis, the turbulence parameters). Secondly, fatigue analyses can be easily adapted to the wind

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**Fig. 8** Functional fits of the first three moments over all wind speeds at the reference turbulence intensity, \( I = 0.16 \)

**Fig. 9** Cycle rates in each of the wind condition bins from the AWT data set

**Fig. 10** Load distributions at various wind speeds estimated from the functional fits to the moments over wind speed

Journal of Solar Energy Engineering
conditions of different sites or certification class designators with this loads model. Recall that wind turbine certification standards are usually tied to a prescribed site characterization or “class.”

**FORM-Based Uncertainty Analysis and Design Load Spectra.** Finally, we show how the foregoing results (e.g., the long-term load distribution in Fig. 11) can be conveniently adjusted to reflect uncertainty in both loading and material behavior. We rely here on concepts from first-order reliability methods (FORM). These provide not only an efficient method to estimate the fatigue reliability of a wind turbine component, but also the particular combination of uncertain factors most likely to cause such failure (the FORM design point).

The program FAROW (Veers et al., 1994) uses FORM methods to propagate uncertainty in 15 different factors; here for simplicity we consider the two (net) uncertain factors, εs and εf, to reflect uncertainty in S-N curve and long-term loads distribution, respectively. The resulting number of cycles to failure, Nfail is

\[ N_{\text{fail}} = \frac{C}{E[S^*]} = \frac{C_{\text{nom}} \cdot e_C}{E[S_{\text{nom}}] \cdot e_s} \]  

(7)

Here \( N = C/S^* \) is the component’s S-N curve, parameterized by a slope \( b \) (fixed) and intercept \( C \) (uncertain). \( S_{\text{nom}} \) and \( C_{\text{nom}} \) include all the factors that influence loading and material resistance, respectively. Ronold et al. (1994) includes a proposed definition of these nominal factors, which will not be repeated here. (Suffice it to say here, \( e_s \) reflects uncertainty in \( S \) due to limited knowledge both of the wind climate distribution—i.e., \( f(V) \) is the probability density of mean wind speed—and of the loads due to limited data at various wind speeds. \( e_C \) reflects uncertainty in material strength and fatigue modeling.)

Numerical routines like FAROW (Veers et al., 1994) could be used here to continue with the uncertainty analysis including the detail needed to accurately reflect the physical situation. To include an analytical solution more fitting for a short example, we here let \( e_C \) and \( e_s \) be assumed to be independent and log-normally distributed. FORM estimates the most likely values to cause failure as

\[ S^* = S_{\text{nom}} \gamma_s; \quad \gamma_s = \exp(+\sigma_{\text{inc}} \alpha_s \beta) \]

\[ C^* = C_{\text{nom}} \gamma_C; \quad \gamma_C = \exp(-\sigma_{\text{inc}} \alpha_C \beta) \]

which are equal to the nominal loading and strength times the safety factors, \( \gamma_s \) and \( \gamma_C \). Here \( \beta = \Phi^{-1}(1 - p_f) \) is the “reliability index” associated with a target failure probability \( p_f \) per service life (\( \Phi^{-1} \) is the inverse Gaussian distribution function).

\[ \alpha_s = b \sigma_{\text{inc}} / \sigma_s \] and \( \alpha_C = \sigma_{\text{inc}} / \sigma_C \), in terms of the net standard deviation of the safety margin \( M_s \):

\[ M_s = \sqrt{(b \sigma_{\text{inc}})^2 + \sigma_{\text{inc}}^2} \]

With the log-normal model, we also have that

\[ \sigma_{\text{inc}} = \sqrt{\ln (1 + \text{COV}^2)} \]

and

\[ \sigma_{\text{inc}} = \sqrt{\ln (1 + \text{COV}^2)} \]

As a numerical example we consider a blade material, with S-N curve characterized by exponent \( b = 6 \), and coefficients of variation \( \text{COV}_s = 0.10 \) and \( \text{COV}_C = 0.50 \), respectively. The above results then yield \( b \sigma_{\text{inc}} = 0.60 \), \( \sigma_s = 0.76 \), and \( \alpha_s = 0.78 \). This gives a load factor \( \gamma_s = 1.2 \) to achieve \( p_f \approx 10^{-2} \) (\( \beta = 2 \)), and \( \gamma_C = 1.3 \) to achieve \( p_f \approx 10^{-3} \) (\( \beta = 3 \)), per service life. These factors can then be applied to a nominal, best-estimated fatigue load spectrum; e.g., by rescaling the long-term distribution in Fig. 11.

Note that this simple, 2-variable formulation was chosen in this example to permit analytical expressions for \( S^* \) and \( C^* \); however, more general FORM codes (e.g., FAROW) provide analogous results, in more complex random variable problems, through numerical optimization routines.

**Conclusions**

Rainflow-counted cyclic-loading amplitudes are described by the first three statistical moments of the amplitudes. Functional forms of these moments are fitted to wind conditions (wind speed and turbulence intensity) by standard regression techniques on the parameters of the functions. The statistics of the regression provide useful information on the nature of the behavior of the loads as a function of wind condition. Plus, the unexplained variation remaining after the regression reflects the degree of uncertainty in the data. The distribution of load amplitudes can then be estimated at any wind speed and used for both fatigue life estimation and overall load spectrum generation. The overall spectrum reflects the wind conditions at a given site or as described in a certification requirement. The uncertainty in the loadings can then be fed into a probabilistic analysis to determine the safety factor required to achieve the desired level of reliability, which is related to the probability of premature failure. All of these features of the moment-based approach to load modeling were illustrated with a specific example.

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